11.2 Homework 1-45 every other odd, 2-44 every other even

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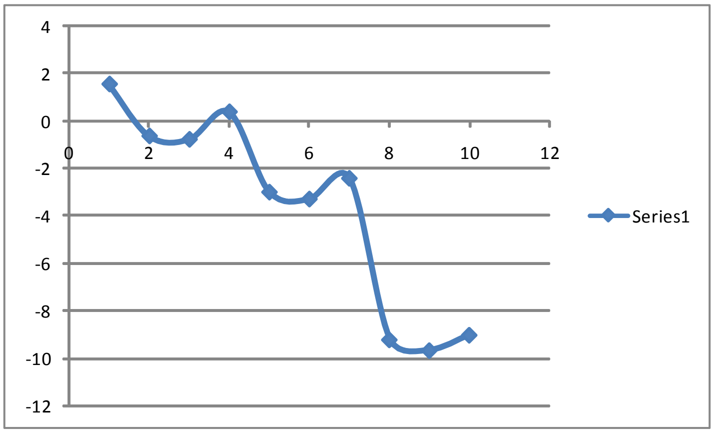
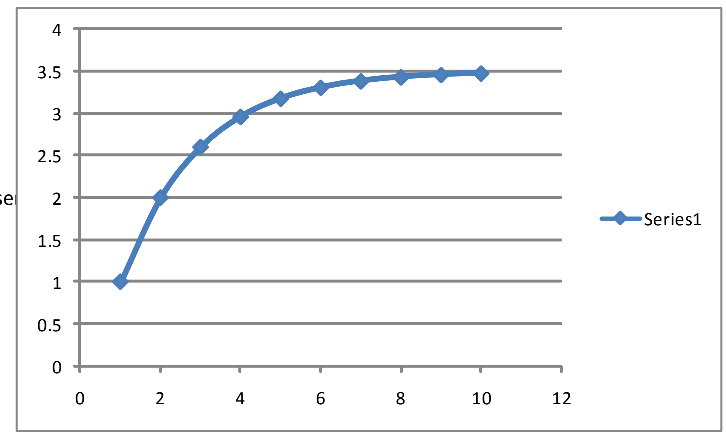
1. .
   a. A sequence is a list of numbers in a particular order. A series is a series of numbers in a particular order added together.
   b. A convergent series when the limit of the series is a real number. It is divergent when the limit is not a real number.

2. The sum of the series from 1 to infinity is 5.

5. .

n	s
1	1.557408
2	-0.62763
3	-0.77018
4	0.387643
5	-2.99287
6	-3.28388
7	-2.41243
8	-9.21214
9	-9.66446
10	-9.0161





Divergent because the se

6. .

n	s
1	1
2	2
3	2.6
4	2.96
5	3.176
6	3.3056
7	3.38336
8	3.430016
9	3.45801
10	3.474806

(a) $a_n = \frac{2n}{3n+1}$

$$\lim_{n\to\infty} \frac{2n}{3n+1} = \lim_{n\to\infty} \frac{\frac{2n}{n}}{\frac{3n}{n}+\frac{1}{n}} = \frac{2}{3}$$

$\{a_n\}$ is convergent

(b) $\sum_{1}^{\infty} a_n$ is divergent because $\lim_{n\to\infty} a_n \neq 0$

9. .

(10) (a) $\sum_{i=1}^{n} a_i$ is the same as $\sum_{j=1}^{n} a_j$

(b) $\sum_{i=1}^{n} a_j = \underbrace{a_j + a_j + a_j \ldots}$ is not the same as $\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots a_n$

n terms

(13) $-2 + \frac{5}{2} - \frac{25}{8} + \frac{125}{32}, \ldots$

$\frac{5/2}{2} = 1.25$

$\boxed{|r| \geq 1 \quad \text{so it is divergent}}$

(14) $1 + 0.4 + 0.16 + 0.064$

$\frac{.4}{1} = .4$

$\boxed{|r| < 1 \quad \text{So it is convergent}}$

(17) $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

$a_1 = \frac{1}{4}$ $\qquad a_3 = \frac{9}{64}$

$a_2 = \frac{-3}{16}$ $\qquad a_4 = \frac{-27}{256}$

$\frac{a_2}{a_1} = -.75$ $\qquad \frac{a_3}{a_2} = -.75$ $\qquad \frac{a_4}{a_3} = -.75$

$\frac{1}{4}\left(1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64}\right)$

$a_1 = \frac{1}{4}(1)$ $a_2 = \frac{1}{4}\left(\frac{-3}{4}\right) = \frac{1}{4}\sum_{n=1}^{\infty}\left(\frac{-3}{4}\right)^{n-1}$

geometric series $\quad a = 1 \quad r = \frac{-3}{4}$

$S = \frac{1}{1 + \frac{3}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7} \cdot \frac{1}{4} = \frac{1}{7}$

$\boxed{\text{converges at } \frac{1}{7}}$

(18) $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ $\qquad a_0 = 1$

$a_1 = \frac{1}{2^{1/2}}$ $\quad a_2 = \frac{1}{2^1}$ $\quad a_3 = \frac{1}{2^{3/2}}$ $\quad a_4 = \frac{1}{2^2}$

$a_0 = 1$ $\quad a_1 = 2^{-1/2}$ $\quad a_2 = 2^{-1}$ $\quad a_3 = 2^{-3/2}$

$a_4 = a^{-4}$

$\frac{a_1}{a_2} = \frac{1}{\sqrt{2}} < 1$ so it $\boxed{\text{converges}}$

$$S = \frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \boxed{2+\sqrt{2}}$$

(21) $\sum_{n=1}^{\infty} \frac{n}{n+5}$

$a_0 = \frac{0}{5}$ $a_1 = \frac{1}{6}$ $a_2 = \frac{2}{7}$ $a_3 = \frac{3}{8}$ $a_4 = \frac{4}{9}$

$\cancel{\frac{\frac{1}{6}}{\frac{2}{7}} = .58\overline{3}}$ not geometric

$$\lim_{n\to\infty} \frac{n}{n+5} = \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{n}{n}+\cancel{\frac{5}{n}}} = 1$$

$\boxed{\text{divergent}}$ because $\lim_{n\to\infty} \frac{n}{n+5} \neq 0$

(22) $\sum_{n=1}^{\infty} \frac{3}{n} = 3\sum_{n=1}^{\infty} \frac{1}{n}$

harmonic series $\boxed{\text{diverges}}$

$a_1 = 1$ $a_2 = \frac{1}{2}$ $a_3 = \frac{1}{3}$ $a_4 = \frac{1}{4}$ ....

(25) $\sum_{k=2}^{\infty} \frac{k^2}{k^2-1}$ $\lim_{k\to\infty} \frac{k^2}{k^2-1} = \lim_{k\to\infty} \frac{1}{1-\cancel{\frac{1}{k^2}}} = 1$

$\lim_{k\to\infty} \frac{k^2}{k^2-1} \neq 0$ so it is $\boxed{\text{divergent}}$

(26) $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = 2\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+1)}$

$$\frac{1}{(n+3)(n+1)} = \frac{A}{n+3} + \frac{B}{n+1}$$

$$1 = A(n+1) + B(n+3)$$

$$1 = An + A + Bn + 3B$$

$$n^0(1) = n(A+B) + n^0(A+3B)$$

$A+B=0$ $A+3B=1$

$A=-B$ $-B+3B=1$

$2B=1$

$A = \frac{-1}{2}$ $B = \frac{1}{2}$

$A = \frac{-1}{2} \qquad B = \frac{1}{2}$

$$2\sum_{n=1}^{\infty} \frac{-\frac{1}{2}}{n+3} + \frac{\frac{1}{2}}{n+1}$$

$$\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+3}$$

$$\underbrace{\left(\frac{1}{2} - \cancel{\frac{1}{4}}\right)}_{a_1} + \underbrace{\left(\frac{1}{3} - \cancel{\frac{1}{5}}\right)}_{a_2} + \underbrace{\left(\cancel{\frac{1}{4}} - \frac{1}{6}\right)}_{a_3} + \underbrace{\left(\cancel{\frac{1}{5}} - \frac{1}{7}\right)}_{a_4}$$

$$\frac{1}{2} + \frac{1}{3} + \lim_{n\to\infty}\left(\frac{1}{n+1} - \frac{1}{n+3}\right)$$

$$\frac{3}{6} + \frac{2}{6} = \boxed{\frac{5}{6} \text{ converges}}$$

29 $\sum_{n=1}^{\infty} \sqrt[n]{2} \qquad \lim_{n\to\infty} 2^{\frac{1}{n}}$

$a_1 = 2 \quad a_2 = \sqrt{2} \quad a_3 = 2^{1/3} \quad a_4 = 2^{1/4}$

$\lim_{n\to\infty} 2^{1/n} = 1 \neq 0$ $\boxed{\text{divergent}}$

30 $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+5}\right)$

$\lim_{n\to\infty} \ln \frac{1}{2+\frac{5}{n}} = \ln\left(\frac{1}{2}\right) \neq 0$ $\boxed{\text{divergent}}$

33 $\sum_{n=1}^{\infty} \left(\frac{3}{n(n+3)} + \frac{5}{4^n}\right)$

$$3\lim_{n\to\infty} \frac{1}{n(n+3)} + 5\lim_{n\to\infty} \frac{1}{4^n}$$

$$\frac{1}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3}$$

$$1 = A(n+3) + B(n)$$

$$n^0(1) = n(A+B) + n^0(3A)$$

$1 = 3A \qquad A+B=0$

$A = \frac{1}{3} \qquad \frac{1}{3}+B=0 \qquad B = -\frac{1}{3}$

$$\lim_{n\to\infty} \frac{1}{n} - \frac{1}{n+3}$$

$$\underbrace{\left(\frac{1}{1} - \cancel{\frac{1}{4}}\right)}_{a_1} + \underbrace{\left(\frac{1}{2} - \cancel{\frac{1}{5}}\right)}_{a_2} + \underbrace{\left(\frac{1}{3} - \cancel{\frac{1}{6}}\right)}_{a_3} + \underbrace{\left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right)}_{a_4} + \underbrace{\left(\cancel{\frac{1}{5}} - \frac{1}{8}\right)}_{a_5} + \underbrace{\left(\cancel{\frac{1}{6}} - \frac{1}{9}\right)}_{a_6} + \underbrace{\left(\cancel{\frac{1}{7}} - \frac{1}{10}\right)}_{a_7}$$

$$\overset{a_1}{\left(\frac{1}{1}-\cancel{\frac{1}{4}}\right)}+\overset{a_2}{\left(\frac{1}{2}-\cancel{\frac{1}{5}}\right)}+\overset{a_3}{\left(\frac{1}{3}-\cancel{\frac{1}{6}}\right)}+\overset{a_4}{\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{7}}\right)}+\overset{a_5}{\left(\cancel{\frac{1}{5}}-\frac{1}{8}\right)}+\overset{a_6}{\left(\cancel{\frac{1}{6}}-\frac{1}{9}\right)}+\overset{a_7}{\left(\cancel{\frac{1}{7}}-\frac{1}{10}\right)}$$

$$1+\frac{1}{2}+\frac{1}{3}+\lim_{n\to\infty}\left(\frac{1}{n}-\frac{1}{n+3}\right)$$

$$\frac{6}{6}+\frac{3}{6}+\frac{2}{6}=\frac{11}{6}$$

geometric series: $\frac{5}{4^n}$

$$a_1=\frac{5}{4}\quad a_2=\frac{5}{16}\quad a_3=\frac{5}{64}\quad a_4=\frac{5}{256}$$

$$\frac{5}{4}\left(\frac{1}{4^0}+\frac{1}{4^1}+\frac{1}{4^2}+\frac{1}{4^3}\right)$$

$$r=\frac{\frac{1}{4}}{1}=\frac{1}{4}$$

$$S=\frac{5/4}{1-\frac{1}{4}}=\frac{5/4}{3/4}=\frac{5}{3}$$

$$\frac{5}{3}+\frac{11}{6}=\frac{21}{6}=\boxed{\frac{7}{2}}$$

34) $\sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)$

geometric $\frac{3}{5^n}$

$$a_1=\frac{3}{5}\quad a_2=\frac{3}{25}\quad a_3=\frac{3}{125}\quad a_4=\frac{3}{625}$$

$$\frac{3}{5}\left(\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}\cdots\right)$$

$$r=\frac{\frac{1}{25}}{\frac{1}{5}}=\frac{1}{25}\cdot\frac{5}{1}=\frac{1}{5}$$

$$S=\frac{3/5}{1-\frac{1}{5}}=\frac{3/5}{4/5}=\frac{3}{5}\cdot\frac{5}{4}=\frac{3}{4}$$

$2\sum_{n=1}^{\infty}\frac{1}{n}$ $\boxed{\text{diverges}}$

37) $3.\overline{417}$

$$3+417\times10^{-3}+417\times10^{-6}+417\times10^{-9}\ldots$$

$$3+417\left[\frac{1}{10^3}+\frac{1}{10^6}+\frac{1}{10^9}\cdots\right]$$

$$3+\frac{417}{10^3}\left[\frac{1}{10^0}+\frac{1}{10^3}\cdots\right]$$

$r = \dfrac{10^{-6}}{10^{-3}} = 10^{-3}$

$S = \dfrac{417\times10^{-3}}{1-10^{-3}} = \dfrac{.417}{.999} = \dfrac{417}{999}$

$3 + \dfrac{417}{999} = \dfrac{3414}{999} = \boxed{\dfrac{1138}{333}}$

(38) $6.2\overline{54}$

$6.2 + .054 + .00054 + .0000054 \ldots$

$6.2 + 54\times10^{-3} + 54\times10^{-5} + 54\times10^{-7} \ldots$

$6.2 + \dfrac{54}{10^3}\left[\dfrac{1}{10^2} + \dfrac{1}{a^4} + \dfrac{1}{a^6} \ldots\right]$

$r = \dfrac{10^{-4}}{10^{-2}} = 10^{-2}$

$S = \dfrac{54\cdot10^{-3}}{1-10^{-2}} = \dfrac{.054}{.99} = \dfrac{54}{990}$

$6.2 + \dfrac{54}{990} = \dfrac{6192}{990} = \dfrac{688}{110} = \boxed{\dfrac{344}{55}}$

(41) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n}$ geometric series

$a_1 = \dfrac{x}{3}$ $a_2 = \dfrac{x^2}{6}$ $a_3 = \dfrac{x^3}{9}$

$r = \dfrac{\frac{x^2}{6}}{\frac{x}{3}} = \dfrac{x}{3}$

$|r|<1$ $\left|\dfrac{x}{3}\right|<1 \to -1 < \dfrac{x}{3} < 1$ or $\boxed{-3 < x < 3}$

$S = \dfrac{x/3}{1-x/3} = \dfrac{x/3}{\frac{3x}{3x} - \frac{x^2}{3x}} = \dfrac{x/3}{\frac{3x-x^2}{3x}}$

$S = \dfrac{\cancel{x}}{3}\cdot\dfrac{3x}{3x-x^2} = \dfrac{3x}{3(3-x)} = \boxed{\dfrac{x}{3-x}}$

$\cancel{x}(3-x)$

(42) $\displaystyle\sum_{n=1}^{\infty} (x-4)^n$ geometric series

$a_1 = x-4$ $a_2 = (x-4)^2 = x^2-8x+16$ $a_3 = (x-4)^3 = x^3-8x^2+16x-4x^2+32x-64$

$= x^3-12x^2+48x-64$

$-1 < \dfrac{(x-4)^2}{x-4} < 1$

$$-1 < (x-4) < 1$$

$$\boxed{3 < x < 5}$$

$$S = \frac{x-4}{1-(x-4)} = \boxed{\frac{x-4}{5-x}}$$

(45) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$ geometric series

$a_0 = 1 \quad a_1 = \frac{\cos x}{2} \quad a_2 = \frac{\cos^2 x}{4} \quad a_3 = \frac{\cos^3 x}{8} \quad a_4 = \frac{\cos^4 x}{16}$

$$r = \frac{\cos x}{2}$$

$$-1 < \frac{\cos x}{2} < 1$$

$$-2 < \cos x < 2$$

$$\boxed{\cos^{-1}(-2) < x < \cos^{-1}(2)}$$

$$S = \frac{1}{1-\cos x/2} = \boxed{\frac{2}{2-\cos x}}$$