11.2 Notes Series

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7:34 PM

1. IntroWe will now add our sequence terms together"

$$\{a_n\} = \{a_1, a_2, a_3 \ldots\ldots a_n\}$$

$$+ \quad + \quad + \; \ldots$$

partial sum

$$S_n = a_1 + a_2 + a_3 \ldots + a_n$$

$$S_3 = a_1 + a_2 + a_3$$

$$S_{10} = a_1 + a_2 + \ldots a_9 + a_{10}$$

sum of all terms

$$a_1 + a_2 + a_3 + \ldots a_n$$

If this sum adds to a finite # we say it converges. We us "S" as its sum.

$$a_1 + a_2 + a_3 \ldots a_n + \ldots$$

$$= \sum_{n=1}^{\infty} a_n$$

$$= S$$

If a series has a limit, s

$$S_1 = a_1$$

$$S_2 = a_1 + a_2 = \sum_{n=1}^{2} a_n$$

$$S_3 = a_1 + a_2 + a_3 \quad \sum_{n=1}^{3} a_n$$

$$S_n = a_1 + a_2 + a_3 \ldots a_{n-1} + a_n = \sum_{n=1}^{N} a_n$$

Technically these partial sum forms another sequence

$$\{S_n\} = \{S_1, S_2, S_3, \ldots S_n, \ldots\}$$

If the sequence has limit "s", the series $\sum_{1}^{\infty} a_n = S$

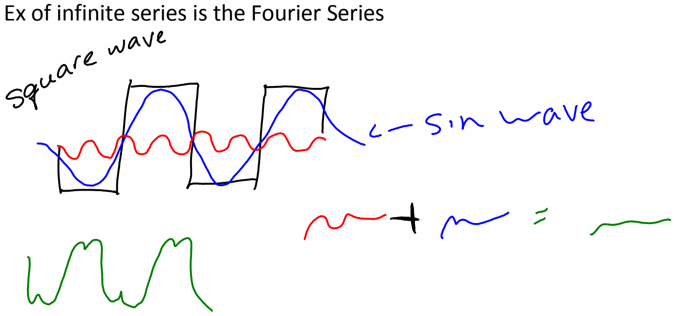
ex) let $a_n = \frac{1}{2n}$ Form a partial sum

$$S_4 = \frac{1}{2.1} + \frac{1}{2 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 4}$$

If this has a limit, and we let it be "s"

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}$$

Ex of infinite series is the Fourier Series



2. Geometric Series

$$a + ar + ar^2 + ar^3 \ldots + ar^{n-1}$$

a = constant = value of 1st term
r = common ratio

ex) $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$

$$a = 5 \quad +\left(\frac{-5 \cdot 2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5 \cdot \left(\frac{-2}{3}\right)^3$$

$$r = \frac{-2}{3}$$

We can analytically find a sum for this infinite series

Partial sum $S_n = a + ar + ar^2 + \ldots . ar^{n-1}$
Multiply by r: $rs_n = ar + ar^2 + ar^3 \ldots . ar^n$
Subtract from eachother: $a - ar^n$
Factor $(1-r) = a(1-r^n)$
$S_n = a(1 - r^n)$

$S_n = a\left(\frac{1-r^n}{1-r}\right)$

$S \stackrel{?}{=} \lim_{n\to\infty} S_n$

$\lim_{n\to\infty} a\left(\frac{1-r^n}{1-r}\right)$

$= \lim_{n\to\infty} \frac{a}{1-r} - \lim_{n\to\infty} \frac{ar^n}{1-r}$

$= \frac{a}{1-r} - \frac{a}{1-r} \lim_{n\to\infty} r^n \leftarrow$ will converge if $|r| < 0$

$S = \frac{a}{1-r} \quad |r| < 0$

ex

#14 $S = 1 + .4 + .16 + 0.064$

$(1)^1 + (.4)^1 + (.4)^2 + (.4)^3$

$a = 1 \qquad r = .4$

$S = \frac{(1)}{1-(.4)} = \frac{1}{.6} = \frac{10}{6} = \frac{5}{3}$

#16 $\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$

$1 + \left(\frac{-6}{5}\right) + \left(\frac{6}{5}\right)^2 + \left(\frac{-6}{5}\right)^3 + \left(\frac{-6}{5}\right)^4 \ldots$

$a = 1$

$r = -\frac{6}{5} \qquad |r| > 1$ divergent

$s = \infty$ divergent

#20)

$$\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = \frac{e}{3^0} + \frac{e^2}{3^1} + \frac{e^3}{3^2} + \frac{e^4}{3^3} \cdots$$

$$e + e\left(\frac{e}{3}\right) + e\left(\frac{e}{3}\right)^2 + e\left(\frac{e}{3}\right)^3$$

$a = e$

$r = e/3 < 1$

$$S = \frac{e}{1-\frac{e}{3}} \cdot \frac{3}{3} = \boxed{\frac{3e}{3-e}} = 28.9$$

Application

$6.231515\overline{15}$ write as # + fraction

$6.23 + .0015 + .000015 + .00000015 \ldots +$

$6.23 + 15\times10^{-4} + 15\times10^{-6} + 15\times10^{-8}$

$$6.23 + 15\left[\frac{1}{10^4} + \frac{1}{10^6} + \frac{1}{10^8} \cdots\right]$$

$$6.23 + \frac{15}{10^4}\left[1 + \frac{1}{10^2} + \frac{1}{10^4} \cdots\right]$$

$$6.23 + \frac{15}{10^4}\left[1 + \left(\frac{1}{10^2}\right)^1 + \left(\frac{1}{10^2}\right)^2 + \left(\frac{1}{10^2}\right)^3 \cdots +\right.$$

$$= \frac{1}{1-\frac{1}{100}} = \frac{1}{\frac{99}{100}} = \frac{100}{99}$$

$$6.23 + \frac{15\cdot100}{10,000\cdot99}$$

$$6.23 + \frac{15}{9900}$$

3. Telescoping series

#26)

$$\sum_{1}^{\infty} \frac{2}{n^2+4n+3}$$

$a_1 = \frac{2}{1+4+3} = \frac{2}{8} = \frac{1}{4}$

$a_2 = \frac{2}{15}$

$a_3 = \frac{1}{12}$

$a_4 = \ldots$

Partial fractions

$$\frac{1}{n^2+4n+3} = \frac{1}{(n+1)(n+3)} = \frac{A}{n+1} + \frac{B}{n+3}$$

$$= \frac{A(n+3) + B(n+1)}{(n+1)(n+3)}$$

$$1 = A(n+3) + B(n+1)$$

$$1 = An + 3A + Bn + B$$

$$1 = n(A+B) + (3A+B)$$

$$1 = 3A + B$$

$$0 = A + B$$

$$A = -B \qquad B = \frac{-1}{2} \quad A = \frac{1}{2}$$

$$\frac{1}{2}\sum_{1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+3}\right)$$

$$= \underset{n=1}{\left(\frac{1}{2} - \cancel{\frac{1}{4}}\right)} + \underset{n=2}{\left(\frac{1}{3} - \cancel{\frac{1}{5}}\right)} + \underset{n=3}{\left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right)} + \underset{n=4}{\left(\cancel{\frac{1}{5}} - \frac{1}{7}\right)} \cancel{\ }$$

$$= \frac{1}{2} + \frac{1}{3}$$

4. Other properties and theorems of infinite series

Theorem: if $\sum_{1}^{\infty} a_n \to L$ then $\{a_n\} \to 0$

if $\sum_{1}^{\infty} a_n = L$ & If $\sum_{1}^{\infty} b_n = M$

(1) $\sum^{\infty} c \cdot a_n = c \cdot \sum a_n = c \cdot L$

(2) $\sum_{1}^{\infty} (a_n \pm b_n) = \sum_{1}^{\infty} a_n \pm \sum_{1}^{\infty} b_n$

$= L \pm M$

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$$\sum_{1}^{\infty}\left(\frac{3}{5^n} + \frac{2}{n}\right)$$

$$= 3\sum_{1}^{\infty}\frac{1}{5^n} + 2\sum_{1}^{\infty}\frac{1}{n} \qquad \text{diverges}$$