

Friday, March 02, 2007
2:41 PM

① $r = \sqrt{\theta} \quad 0 \leq \theta \leq \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} (\sqrt{\theta})^2 d\theta$$

$$\frac{1}{2} \int_0^{\frac{\pi}{4}} \theta d\theta$$

$$\frac{1}{2} \left(\frac{\theta^2}{2} \right) \bigg|_0^{\frac{\pi}{4}} = \frac{1}{4} \left(\frac{1}{4} \cdot \frac{\pi^2}{4} \right) = \frac{\pi^2}{64}$$

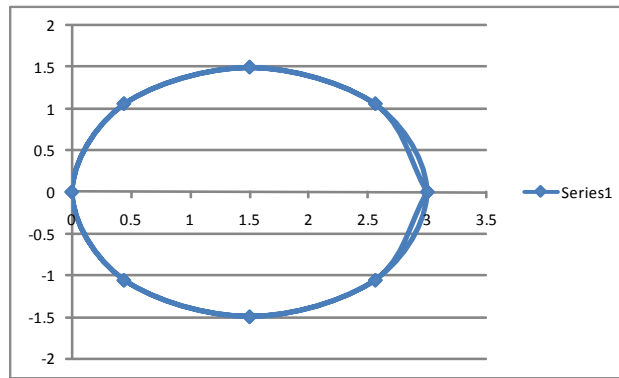
⑤ $r = \theta \quad 0 \leq \theta \leq \pi$

$$\frac{1}{2} \int_0^{\pi} \theta^2 d\theta$$

$$\frac{1}{2} \left(\frac{\theta^3}{3} \right) \bigg|_0^{\pi} = \frac{\pi^3}{6}$$

⑨ $r = 3 \cos \theta$

theta	r=3*cos(theta)	x	y
0	3	3	0
0.392699	2.771639	2.560661	1.060659
0.785398	2.121322	1.500002	1.5
1.178096	1.148053	0.439342	1.060662
1.570795	3.98E-06	5.28E-12	3.98E-06
1.963494	-1.14805	0.439336	-1.06066
2.356193	-2.12132	1.499994	-1.5
2.748891	-2.77164	2.560655	-1.06067
3.14159	-3	3	-8E-06
3.534289	-2.77164	2.560667	1.060654
3.926988	-2.12133	1.50001	1.5
4.319686	-1.14806	0.439348	1.060668
4.712385	-1.2E-05	4.75E-11	1.19E-05
5.105084	1.148038	0.439331	-1.06065
5.497783	2.12131	1.499986	-1.5
5.890481	2.771633	2.56065	-1.06067
6.28318	3	3	-1.6E-05



$$\frac{1}{2} \int_0^{2\pi} 9 \cos^2 \theta d\theta$$

$$\frac{9}{2} \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} d\theta = \frac{9}{4} \int_0^{2\pi} 1 + \cos 2\theta d\theta \quad \text{let } u = 2\theta \quad du = 2 d\theta$$

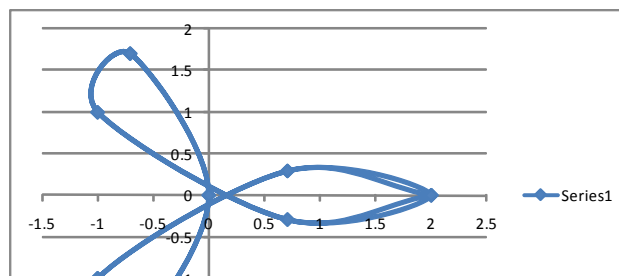
$$\frac{9}{4} (2\pi) + \frac{9}{8} \int \cos u du$$

$$\frac{9\pi}{4} + \left(\frac{9}{8} \sin 2\theta \right) \bigg|_0^{2\pi}$$

$$\frac{9\pi}{4} - 0 = \frac{9\pi}{4}$$

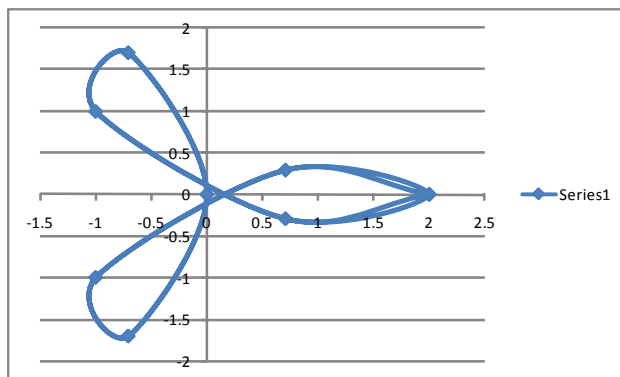
⑬ $r = 2 \cos 3\theta$

theta	r=2*cos(3*theta)	x	y
0	2	2	0
0.392699	0.765369	0.707109	0.292894
0.785398	-1.41421	-1	-1
1.178096	-1.84776	-0.70711	-1.70711
1.570795	-8E-06	-1.1E-11	-8E-06



⑮) $r = 2\cos 3\theta$

theta	r=2cos3(theta)	x	y
0	2	2	0
0.392699	0.765369	0.707109	0.292894
0.785398	-1.41421	-1	-1
1.178096	-1.84776	-0.70711	-1.70711
1.570795	-8E-06	-1.1E-11	-8E-06
1.963494	1.847755	-0.7071	1.707104
2.356193	1.414222	-1	1.000008
2.748891	-0.76535	0.707094	-0.29289
3.14159	-2	2	-5.3E-06
3.534289	-0.76538	0.707123	0.292897
3.926988	1.414199	-0.99999	-0.99999
4.319686	1.847767	-0.70712	-1.70711
4.712385	2.39E-05	-9.5E-11	-2.4E-05
5.105084	-1.84775	-0.7071	1.707101
5.497783	-1.41423	-1.00001	1.000019
5.890481	0.765339	0.70708	-0.29289
6.28318	2	2	0



$$\frac{1}{2} \int_0^{2\pi} 2 \cos^2 3\theta d\theta$$

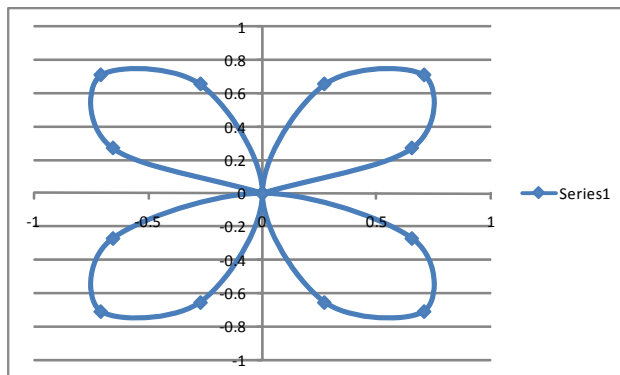
$$\int_0^{2\pi} \frac{1 + \cos 2\theta}{2} d\theta = \int_0^{2\pi} \frac{1}{2} d\theta + \int_0^{2\pi} \frac{\cos 2\theta}{2} d\theta$$

$$\frac{1}{2} \theta \Big|_0^{2\pi} + \frac{1}{2} \sin 2\theta \Big|_0^{2\pi}$$

π

⑰) $r = \sin 2\theta$

theta	r=sin(2theta)	x	y
0	0	0	0
0.392699	0.707106	0.653281	0.270598
0.785398	1	0.707107	0.707106
1.178096	0.707108	0.270599	0.653283
1.570795	2.65E-06	3.52E-12	2.65E-06
1.963494	-0.7071	0.270596	-0.65328
2.356193	-1	0.707105	-0.70711
2.748891	-0.70711	0.653284	-0.2706
3.14159	-5.3E-06	5.31E-06	-1.4E-11
3.534289	0.707103	-0.65328	-0.27059
3.926988	1	-0.70711	-0.7071
4.319686	0.707112	-0.2706	-0.65329
4.712385	7.96E-06	-3.2E-11	-8E-06
5.105084	-0.7071	-0.27059	0.653277
5.497783	-1	-0.7071	0.70711
5.890481	-0.70711	-0.65329	0.270604
6.28318	-1.1E-05	-1.1E-05	5.63E-11



$$\frac{1}{2} \int_0^{\pi/2} \sin^2 2\theta d\theta$$

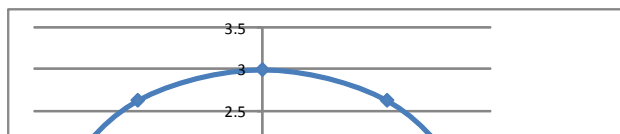
$$\frac{1}{2} \int_0^{\pi/2} \frac{1 - \cos 4\theta}{2} d\theta = \frac{1}{4} \int_0^{\pi/2} (1 - \cos 4\theta) d\theta$$

$$\frac{\pi}{8} - \left(\frac{1}{8} \sin 4\theta \right) \Big|_0^{\pi/2}$$

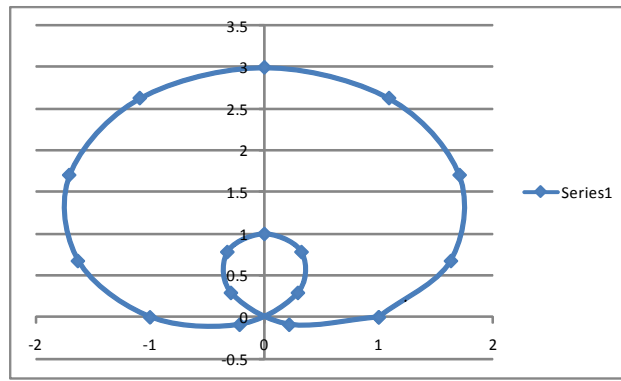
$\frac{\pi}{8}$

⑳) $r = 1 + 2\sin \theta$ (inner loop)

theta	r=1+2sin(theta)	x	y
0	1	1	0



0.392699	1.765366	1.630986	0.675576
0.785398	2.414213	1.707107	1.707105
1.178096	2.847758	1.089793	2.630985
1.570795	3	3.98E-06	3
1.963494	2.84776	-1.08979	2.630989
2.356193	2.414216	-1.70711	1.707112
2.748891	1.765371	-1.63099	0.675582
3.14159	1.000005	-1.00001	2.65E-06
3.534289	0.234639	-0.21678	-0.08979
3.926988	-0.41421	0.292891	0.292889
4.319686	-0.84776	0.324425	0.783223
4.712385	-1	3.98E-06	1
5.105084	-0.84776	-0.32442	0.783232
5.497783	-0.41422	-0.2929	0.292899
5.890481	0.234624	0.216764	-0.08979
6.28318	0.999989	0.999989	-5.3E-06



$$A = 2 \int_{-\pi/2}^{\pi/2} \frac{(1+2\sin\theta)^2}{2} d\theta = \int_{-\pi/2}^{\pi/2} (1+4\sin\theta+4\sin^2\theta) d\theta$$

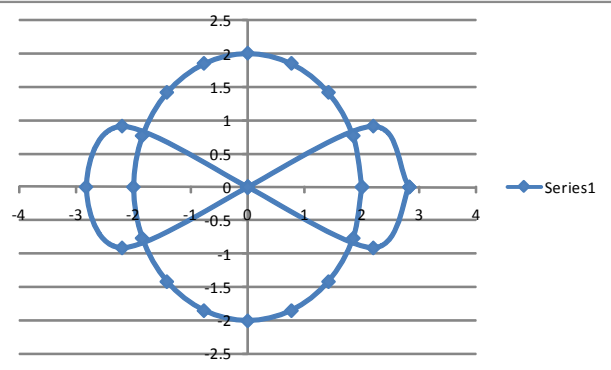
$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\begin{aligned} & \theta \Big|_{-\pi/2}^{\pi/2} - (4\cos\theta) \Big|_{-\pi/2}^{\pi/2} + 4 \int_{-\pi/2}^{\pi/2} \frac{(1-\cos 2\theta)}{2} d\theta \\ & \frac{\pi}{2} + \frac{\pi}{2} - 4(\cos(-\frac{\pi}{2}) - \cos(\frac{\pi}{2})) + 2 \left[\theta - \frac{\sin 2\theta}{2} \right] \\ & -\frac{\pi}{2} + \frac{\pi}{2} - 4(\sqrt{\frac{3}{2}}) + 2 \left(-\frac{\pi}{2} + \frac{\pi}{2} \right) - \left(\frac{\sin(-\frac{2\pi}{2})}{2} - \frac{\sin(\frac{2\pi}{2})}{2} \right) \\ & \frac{\pi}{3} - 2\sqrt{3} + 2 \left(\frac{\pi}{3} + \frac{\pi}{3} \right) \\ & \pi - 2\sqrt{3} + \sqrt{3}/2 = \boxed{\frac{\pi - 3\sqrt{3}}{2}} \end{aligned}$$

②⑤ $r^2 = 8\cos 2\theta$, $r = 2$

theta	r=sqrt(8cos 2theta)	x	y
0	2.828427	2.828427	0
0.392699	2.378415	2.197369	0.910179
0.785398	0.003258	0.002304	0.002304
1.178096	#NUM!	#NUM!	#NUM!
1.570795	#NUM!	#NUM!	#NUM!
1.963494	#NUM!	#NUM!	#NUM!
2.356193	#NUM!	#NUM!	#NUM!
2.748891	2.378409	-2.19736	0.910183
3.14159	2.828427	-2.82843	7.51E-06
3.534289	2.378421	-2.19738	-0.91018
3.926988	0.007285	-0.00515	-0.00515
4.319686	#NUM!	#NUM!	#NUM!
4.712385	#NUM!	#NUM!	#NUM!
5.105084	#NUM!	#NUM!	#NUM!
5.497783	#NUM!	#NUM!	#NUM!
5.890481	2.378402	2.197353	-0.91019
6.28318	2.828427	2.828427	-1.5E-05

theta	r=2	x	y
0	2	2	0
0.392699	2	1.847759	0.765366
0.785398	2	1.414215	1.414213
1.178096	2	0.765369	1.847758
1.570795	2	2.65E-06	2
1.963494	2	-0.76536	1.84776
2.356193	2	-1.41421	1.414216
2.748891	2	-1.84776	0.765371
3.14159	2	-2	5.31E-06
3.534289	2	-1.84776	-0.76536
3.926988	2	-1.41422	-1.41421
4.319686	2	-0.76537	-1.84776
4.712385	2	-8E-06	-2
5.105084	2	0.765359	-1.84776
5.497783	2	1.414207	-1.41422
5.890481	2	1.847755	-0.76538
6.28318	2	2	-1.1E-05

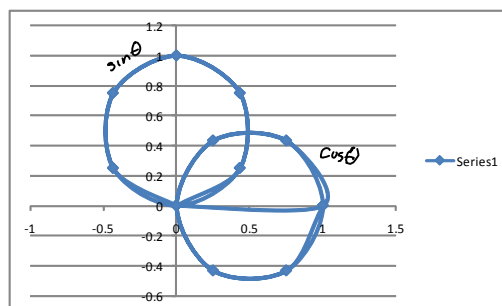


$$\frac{12\pi}{6} - \frac{\pi}{6} \sim \frac{11\pi}{6}$$

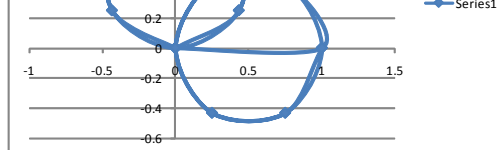
$$\begin{aligned} \sqrt{8\cos 2\theta} &= 2 \\ 8\cos 2\theta &= 4 \\ \cos 2\theta &= \frac{1}{2} \quad \theta = \frac{\pi}{6}, -\frac{\pi}{6}, \frac{11\pi}{6}, -\frac{11\pi}{6} \\ A &= 2 \int_{\pi/6}^{11\pi/6} 4^2 d\theta = 32\theta \Big|_{\pi/6}^{11\pi/6} = \frac{352\pi}{6} - \frac{32\pi}{6} = \boxed{\frac{160\pi}{3}} \end{aligned}$$

②⑨ $r = \sin\theta$, $r = \cos\theta$

$$\sin\theta = \cos\theta$$



$$\theta = \frac{\pi}{4}, 0$$



Inside region: $A = \frac{1}{2} \int_0^{\pi/4} \sin^2 \theta - \cos^2 \theta d\theta$

$$= \frac{1}{2} \int \frac{1 - \cos 2\theta}{2} d\theta - \frac{1}{2} \int \frac{1 + \cos 2\theta}{2} d\theta$$

$$\frac{1}{4} \int 1 - \cos 2\theta d\theta - \frac{1}{4} \int 1 + \cos 2\theta d\theta$$

$$\frac{1}{4} \left(\theta - \frac{\sin 2\theta}{2} - \theta - \frac{\sin 2\theta}{2} \right) \Big|_0^{\pi/4}$$

$$\frac{1}{4} \left[\left(\frac{\pi}{4} - \frac{1}{2} - \frac{\pi}{4} - \frac{1}{2} \right) - (0) \right]$$

$$-\frac{1}{8} - \frac{1}{8} = -\frac{1}{4}$$

Entire region $A = \frac{1}{2} \int_0^{\pi} \sin^2 \theta d\theta + \frac{1}{2} \int_0^{\pi} \cos^2 \theta d\theta$

$$\frac{1}{2} \int \frac{1 - \cos 2\theta}{2} d\theta + \frac{1}{2} \int \frac{1 + \cos 2\theta}{2} d\theta$$

$$\frac{1}{4} \left[\theta - \frac{\sin 2\theta}{2} + \theta + \frac{\sin 2\theta}{2} \right] \Big|_0^{\pi}$$

$$\frac{1}{4} (2\theta) \Big|_0^{\pi} = \frac{\pi}{2}$$

Area = Entire region - inside region

$$\frac{\pi}{2} - \frac{1}{4}$$

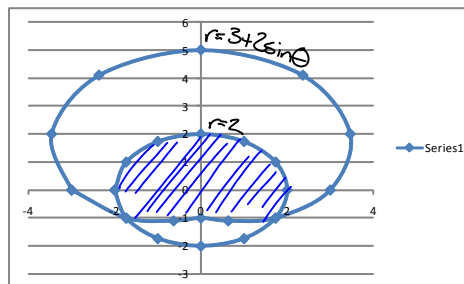
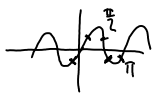
(33) $r = 3 + 2\sin \theta$, $r = 2$

$$2 = 3 + 2\sin \theta$$

$$-1 = 2\sin \theta$$

$$-\frac{1}{2} = \sin \theta$$

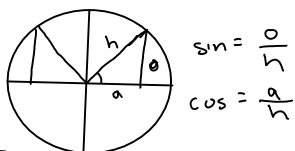
$$\theta = \frac{11\pi}{6}, \frac{7\pi}{6}$$



(37) $r = \sin \theta$, $r = \cos \theta$



$$\frac{\pi}{4}, \frac{3\pi}{4}$$



$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

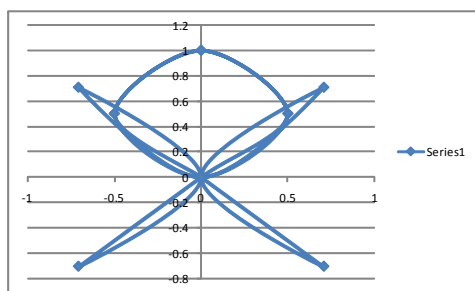
$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4} \right) \text{ and } \left(-\frac{\sqrt{2}}{2}, \frac{5\pi}{4} \right)$$

41) $r = \sin(\theta)$ $r = \sin(2\theta)$ $(r, \theta) = \left(\frac{\sqrt{3}}{2}, \frac{\pi}{3} \right)$

$$\sin \theta = \sin 2\theta$$

$$\theta = 2\theta \quad \theta = 0, \frac{\pi}{3}, \frac{2\pi}{3} = \left(0, 0 \right) \quad \left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3} \right)$$



45) $r = 3\sin \theta$, $0 \leq \theta \leq \frac{\pi}{3}$

$$\frac{dr}{d\theta} = 3\cos \theta$$

$$L = \int_0^{\pi/3} \sqrt{(3\sin \theta)^2 + (3\cos \theta)^2} d\theta$$

$$L = \int_0^{\pi/3} \sqrt{9\sin^2 \theta + 9\cos^2 \theta} d\theta$$

$$L = \int_0^{\pi/3} 3\sqrt{1} d\theta$$

$$3\theta \Big|_0^{\pi/3} = \pi$$

46) $r = e^{2\theta}$ $0 \leq \theta \leq 2\pi$

$$\frac{dr}{d\theta} = 2\theta e^{2\theta}$$

$$L = \int_0^{2\pi} \sqrt{(e^{2\theta})^2 + (2\theta e^{2\theta})^2} d\theta$$

$$\int_0^{2\pi} \sqrt{e^{4\theta} + 4\theta^2 e^{4\theta}} d\theta$$

$$\int_0^{2\pi} \sqrt{e^{4\theta} (1 + 4\theta^2)} d\theta$$

47) $r = \theta^2$, $0 \leq \theta \leq 2\pi$

$$\frac{dr}{d\theta} = 2\theta$$

$$L = \int_0^{2\pi} \sqrt{\theta^4 + 4\theta^2} d\theta$$

(48) $r = \theta \quad 0 \leq \theta \leq 2\pi$

$$\frac{dr}{d\theta} = 1$$

$$L = \int_0^{2\pi} \sqrt{1 + \theta^2} d\theta$$

$$\text{let } \theta = \tan x \quad d\theta = \sec^2 x dx$$

$$= \int \sqrt{1 + \tan^2 x} \sec^2 x dx$$

$$\int \sec^2 x \cdot \sec^2 x dx = \int \sec^4 x dx$$

$$= \frac{1}{3} \tan x \sec^2 x + \frac{2}{3} \int \sec^2 x dx$$

$$x = \tan^{-1} \theta \quad \left(\frac{1}{3} \theta \sec^2(\tan^{-1} \theta) + \frac{2}{3} \theta \right) \Big|_0^{2\pi}$$



$$\tan^{-1} = \frac{\cos}{\sin}$$

$$\frac{2\pi \cdot 6.36 + \frac{4\pi}{3}}{3}$$

$$\frac{0}{h} \cdot \frac{a}{h}$$