HW 12.4 1-33 odd

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8:31 PM

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① $a = \langle 1,2,0 \rangle \qquad b = \langle 0,3,1 \rangle$

$$\left|\begin{matrix} i & j & k \\ 1 & 2 & 0 \\ 0 & 3 & 1 \end{matrix}\right| \begin{matrix} i & j \\ 1 & 2 \\ 0 & 3 \end{matrix}$$

$-(0k + 0i + 1j) + (2i + 0j + 3k)$

$2i - j + 3k$

$$\boxed{\vec{a} \times \vec{b} = \langle 2,-1,3 \rangle}$$

$\cos\theta = \dfrac{\vec{a} \cdot (\vec{a \times b})}{|\vec{a}||\vec{a \times b}|}$ where $\theta \stackrel{?}{=} 90^\circ$

$\cos\theta = \dfrac{2-2+0}{\sim\sim\sim} = 0$

$\boxed{\theta = 90 \checkmark}$

$\cos\theta = \dfrac{\vec{b} \cdot (\vec{a \times b})}{|\vec{b}||(\vec{a \times b})|}$ where $\theta \stackrel{?}{=} 90^\circ$

$\cos\theta = \dfrac{0-3+3}{\sim\sim} = 0$

$\boxed{\theta = 90^\circ \checkmark}$

Yes, the cross product $\langle 2,-1,3 \rangle$ is orthogonal to $\vec{a}$ and $\vec{b}$.

③ $a = 2i + j - k \qquad b = j + 2k$

$2i + 0j + 2k - (0k - 1i + 4j) = 3i - 4j + 2k$

$$\left|\begin{matrix} i & j & k \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{matrix}\right| \begin{matrix} i & j \\ 2 & 1 \\ 0 & 1 \end{matrix}$$

$$\boxed{\vec{a} \times \vec{b} = \langle 3,-4,2 \rangle}$$

$\cos\theta = \dfrac{\vec{a} \cdot (\vec{a \times b})}{|\vec{a}||\vec{a \times b}|} = \dfrac{6-4-2}{\sim\sim} = 0$

$\boxed{\theta = 90^\circ \checkmark}$

$\cos\theta = \vec{b} \cdot (\vec{a \times b}) = \underline{0-4+4} = 0$

$$\frac{}{|\vec{b}||\vec{a\times b}|} \qquad \frac{}{\sim\sim}$$

$\theta = 90^\circ$ ✓

⑤ $a = 3i + 2j + 4k = \langle 3, 2, 4\rangle \quad b = i - 2j - 3k = \langle 1, -2, -3\rangle$

$-6i + 4j - 6k - (2k - 8i - 9j) = 2i + 13j - 8k$

$$\left|\begin{array}{ccc} i & j & k \\ 3 & 2 & 4 \\ 1 & -2 & -3 \end{array}\right|\begin{array}{cc} i & j \\ 3 & 2 \\ 1 & -2 \end{array}$$

$\vec{a} \times \vec{b} = \langle 2, 13, -8\rangle$

$$\cos\theta = \frac{\vec{a}\cdot(\vec{a\times b})}{|\vec{a}||\vec{a\times b}|} = \frac{6 + 26 - 32}{\sim\sim\sim} = 0$$

$\theta = 90^\circ$ ✓

$$\cos\theta = \frac{\vec{b}\cdot(\vec{a\times b})}{|\vec{a}||\vec{a\times b})} = \frac{2 - 26 + 24}{\sim\sim} = 0$$

$\theta = 90^\circ$ ✓

⑦ $a = \langle t, t^2, t^3\rangle \quad b = \langle 1, 2t, 3t^2\rangle$

$3t^4 i + t^3 j + 2t^2 k - (t^3 k + 2t^4 i + 3t^3 j) = t^4 i - 2t^3 j + t^2 k$

$$\left|\begin{array}{ccc} i & j & k \\ t & t^2 & t^3 \\ 1 & 2t & 3t^2 \end{array}\right|\begin{array}{cc} i & j \\ t & t^2 \\ 1 & 2t \end{array}$$

$\vec{a} \times \vec{b} = \langle t^4, -2t^3, t^2\rangle$

$$\cos\theta = \frac{\vec{a}\cdot(\vec{a\times b})}{|\vec{a}||\vec{a\times b}|} = \frac{t^5 - 2t^5 + t^5}{\sim\sim} = 0$$

$\theta = 90^\circ$ ✓

$$\cos\theta = \frac{\vec{b}\cdot(\vec{a\times b})}{|\vec{b}||\vec{a\times b}|} = \frac{t^4 - 4t^4 + 3t^4}{\sim\sim\sim} = 0$$

$\theta = 90^\circ$ ✓

⑨ ⓐ scalar ⓑ meaningless ⓒ vector

ⓓ meaningless ⓔ meaningless ⓕ scalar

⑪ $|u| = 6 \quad |v| = 8$

$$|u \times v| = (6)(8)\sin(150^\circ)$$

$$\boxed{|u \times v| = 24, \text{ into the page}}$$

(13) $a = \langle 3,1,2 \rangle \quad b = \langle -1,1,0 \rangle \quad c = \langle 0,0,-4 \rangle$

$$a \times (b \times c) \neq (a \times b) \times c$$

$$b \times c = -4i + 0j + 0k - (1k + 0i + 4j) = \langle -4,-4,-1 \rangle$$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ -1 & 1 & 0 & -1 & 1 \\ 0 & 0 & -4 & 0 & 0 \end{array}\right.$$

$$a \times (b \times c) = -1i - 8j - 12k - (-4k - 8i - 3j) = \langle 7,-5,-8 \rangle$$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 3 & 1 & 2 & 3 & 1 \\ -4 & -4 & -1 & -4 & -4 \end{array}\right.$$

$$a \times b = 0i - 2j + 3k - (-1k + 2i + 0j) = \langle 1,-2,4 \rangle$$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 3 & 1 & 2 & 3 & 1 \\ -1 & 1 & 0 & -1 & 1 \end{array}\right.$$

$$c \times (a \times b) = 0i - 4j + 0k - (0k + 8i + 0j) = \langle -8,-4,0 \rangle$$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 0 & -4 & 0 & 0 \\ 1 & -2 & 4 & 1 & -2 \end{array}\right.$$

$$\boxed{\langle 7,-5,-8 \rangle \neq \langle -8,-4,0 \rangle}$$

(15) $\langle 1,-1,1 \rangle \times \langle 0,4,4 \rangle = -4i + 0j + 4k - (0k + 4i + 4j) = \langle -8,-4,4 \rangle$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & -1 & 1 & 1 & -1 \\ 0 & 4 & 4 & 0 & 4 \end{array}\right.$$

$$\boxed{-8i - 4j + 4k}$$

(17) let $\vec{a} = \langle 1,1,1 \rangle$

$a \times 0 \quad 0 - 0 = 0 \qquad\qquad 0 \times a = \quad 0 - 0 = 0$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & 1 & 1 & 1 & 1 \end{array}\right. \qquad \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 0 & 0 & 0 & 0 \end{array}\right.$$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}\begin{matrix} i & j \\ 1 & 1 \\ 0 & 0 \end{matrix} \qquad \begin{vmatrix} i & j & k \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}\begin{matrix} i & j \\ 0 & 0 \\ 1 & 1 \end{matrix}$$

$\boxed{\text{So } 0 \times a = 0 = 0 \times a}$

(19) $a \times b = -b \times a$

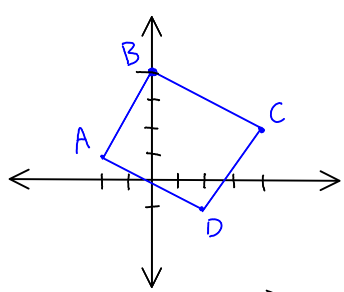
$a \times b = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle$

$(a_2b_3 - a_3b_2)i + (a_3b_2 - a_1b_3)j + (a_1b_2 - a_2b_1)k$

$-b \times a = (a_3b_2 - a_2b_3)i + (a_1b_3 + a_3b_2)j + (a_2b_1 - a_1b_2)k$

(21) $a \times (b + c) = a \times b + a \times c$

(23) $A(-2,1)$ $B(0,4)$ $C(4,2)$ $D(2,-1)$



$\vec{AD} = \langle -4, 2 \rangle$ $\qquad \vec{AB} = \langle -2, -3 \rangle$

$\vec{AD} \times \vec{AB} = \begin{vmatrix} -4 & 2 \\ -2 & -3 \end{vmatrix} = 12 + 4 = \boxed{16}$

(25) $P(1,0,0)$ $Q(0,2,0)$ $R(0,0,3)$

(a) $\vec{PQ} = \langle 1, -2, 0 \rangle$ $\qquad \vec{PR} = \langle 1, 0, -3 \rangle$

$$\overrightarrow{PQ}\times\overrightarrow{PR} = 6i+0j+0k-(-2k+0i-3j) = \boxed{\langle 6,3,2\rangle}$$

$$\begin{vmatrix} i & j & k \\ 1 & -2 & 0 \\ 1 & 0 & -3 \end{vmatrix}\begin{matrix} i & j \\ 1 & -2 \\ 1 & 0 \end{matrix}$$

(b) $$\frac{|\overrightarrow{PQ}\times\overrightarrow{PR}|}{2} = \frac{\sqrt{36+9+4}}{2} = \boxed{\frac{7}{2}}$$

27) $P(0,-2,0)$ $Q(4,1,-2)$ $R(5,3,1)$

(a) $\overrightarrow{PQ} = \langle -4,-3,2\rangle$ $\overrightarrow{PR} = \langle -5,-5,-1\rangle$

$$\overrightarrow{PQ}\times\overrightarrow{PR} = 3i-10j+20k-(15k-10i+4j) = \boxed{\langle 13,-14,5\rangle}$$

$$\begin{vmatrix} i & j & k \\ -4 & -3 & 2 \\ -5 & -5 & -1 \end{vmatrix}\begin{matrix} i & j \\ -4 & -3 \\ -5 & -5 \end{matrix}$$

(b) $$\frac{|\overrightarrow{PQ}\times\overrightarrow{PR}|}{2} = \frac{\sqrt{169+196+25}}{2} = \boxed{\frac{\sqrt{390}}{2}}$$

29) $a=\langle 6,3,-1\rangle$ $b=\langle 0,1,2\rangle$ $c=\langle 4,-2,5\rangle$

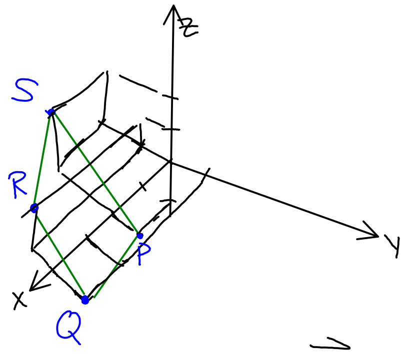
$$a\cdot(b\times c) = \begin{vmatrix} 6 & 3 & -1 \\ 0 & 1 & 2 \\ 4 & -2 & 5 \end{vmatrix}$$

$$6\begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} - 3\begin{vmatrix} 0 & 2 \\ 4 & 5 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 4 & -2 \end{vmatrix}$$

$$6(5+4)-3(0-8)-1(0-4)$$

$$54+24+4 = \boxed{82}$$

31) $P(2,0,-1)$ $Q(4,1,0)$ $R(3,-1,1)$ $S(2,-2,2)$



$\vec{PQ} = \langle -2,-1,-1\rangle \qquad \vec{QR}\ \langle 1,2,-1\rangle \qquad \vec{QS} = \langle 2,3,-2\rangle$

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -2 \\ -2 & -1 & -1 \end{vmatrix}$$

$$1\begin{vmatrix} 3 & -2 \\ -1 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & -2 \\ -2 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ -2 & -1 \end{vmatrix}$$

$$1(-3-2) - 2(-3-4) - 1(-2+6)$$

$$5 + 14 - 4 = \boxed{15}$$

(33) $a = 2i + 3j + k = \langle 2,3,1\rangle \quad b = i - j = \langle 1,-1,0\rangle \quad c = 7i + 3j + 2k = \langle 7,3,2\rangle$

$$\begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 0 \\ 7 & 3 & 2 \end{vmatrix} = 2\begin{vmatrix} -1 & 0 \\ 3 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 7 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 7 & 3 \end{vmatrix}$$

$$= 2(-2-0) - 3(2-0) + 1(3+7)$$

$$-4 - 6 + 10 = \boxed{0}$$

yes coplanar