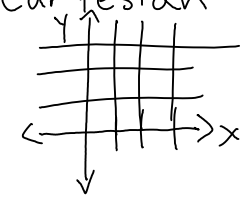
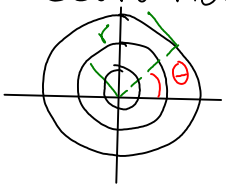
Notes: 10.3 Polar Coordinates

Tuesday, February 27, 2007
8:21 PM

Cartesian Coordinates

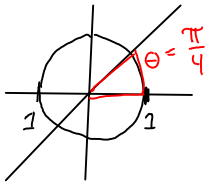


Polar Coordinates

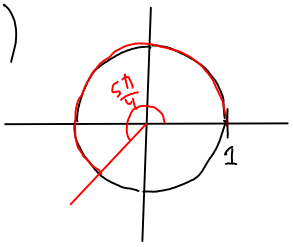


(r, θ)

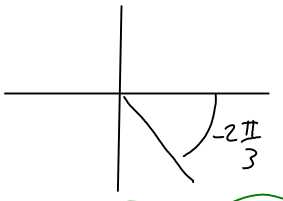
ex (a) $(1, \frac{\pi}{2})$

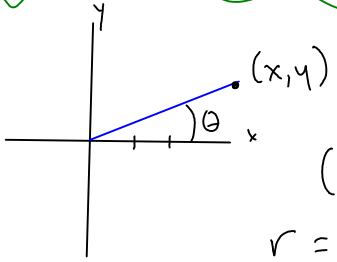


(b) $(1, \frac{5\pi}{4})$



$(2, -\frac{2\pi}{3})$





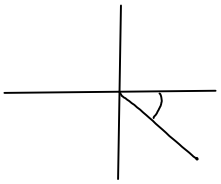
(r, θ)

$r = \sqrt{x^2 + y^2}$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

ex



$(x, y) = (1, -1)$

$(r, \theta) = ?$

$r = \sqrt{(1)^2 + (-1)^2}$

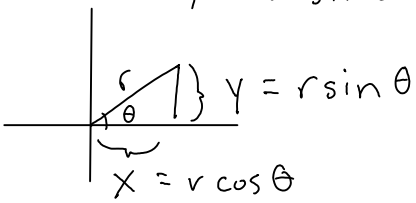
$= \sqrt{2}$

$\theta = \tan^{-1}(-1)$

$(r, \theta) = (2, \frac{\pi}{3}) \rightarrow (x, y) = (1, \sqrt{3})$

$x = 2 \cdot \cos\frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1$

$y = 2 \cdot \sin\frac{\pi}{3} = 2\frac{\sqrt{3}}{2} = \sqrt{3}$



ex given $x = R\cos\theta$ $y = R\sin\theta$

write the eqn in cartesian coords

$\frac{x}{R} = \cos\theta$ $\cos^2 + \sin^2 = 1$

$\frac{y}{R} = \sin\theta$ $\left(\frac{x}{R}\right)^2 + \left(\frac{y}{R}\right)^2 = 1 = x^2 + y^2 = R^2$

backwards

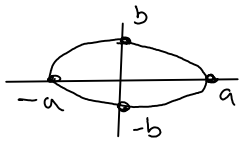
$x^2 + y^2 = R^2$

$(r\cos\theta)^2 + (r\sin\theta)^2 = R^2$

$r^2 \underbrace{(\cos^2\theta + \sin^2\theta}_{=1} = R^2$

$r = R$

ex $\left(\frac{x^2}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$



$$\left(\frac{r\cos\theta}{a}\right)^2 + \left(\frac{r\sin\theta}{b}\right)^2 = 1$$

$$r^2\left[\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right] = 1$$

$$r\theta = \frac{1}{\sqrt{\frac{\cos^2}{a^2} + \frac{\sin^2\theta}{b^2}}}$$

10.3 #56 $r(\theta) = 2 - \sin\theta$ slope at $\theta = \frac{\pi}{3}$

$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta + r\sin\theta}$ $\qquad r'\theta = -\cos\theta$

$$= \frac{(-\cos\theta)\sin\theta + (2-\sin\theta)\cos\theta}{-\cos\theta\cos\theta - (2-\sin\theta)\sin\theta}$$