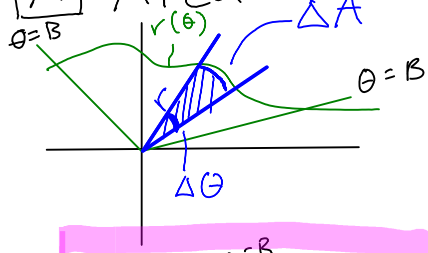
Notes: 10.4 Areas and Lengths in Polar Coordinates

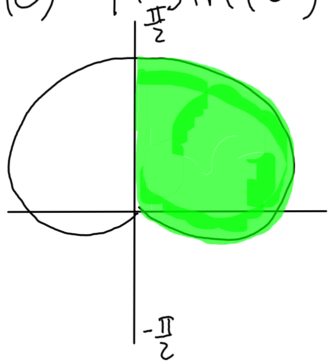
Thursday, March 01, 2007
9:00 PM

$\boxed{A}$ Area



$$\boxed{A = \frac{1}{2}\int_{\theta=A}^{\theta=B} r^2\theta \, d\theta}$$

$\boxed{\text{Ex}}$ $r(\theta) = 1 + \sin(\theta)$



$$A = \int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}} \frac{1}{2}\left(1+\sin(\theta)\right)^2 d\theta$$

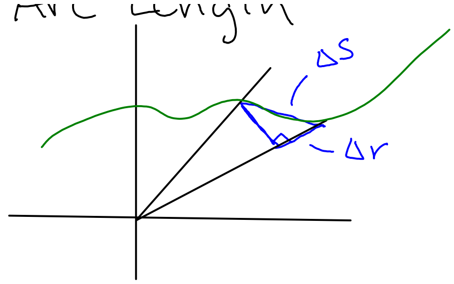
$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + 2\sin\theta + \sin^2\theta \, d\theta$$

$$\frac{1}{2}\left(\theta - 2\cos\theta + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \cos 2\theta)\, d\theta\right)$$

$$\frac{3\pi}{4}$$

$\boxed{B}$ Arc Length

B] Arc Length



$$\Delta s = \sqrt{(r\Delta\theta)^2 + (\Delta r)^2}$$

$$\boxed{\int_{\theta=\alpha}^{\theta=\beta} \sqrt{r^2\theta + \left(\frac{dr}{d\theta}\right)^2}\, d\theta}$$

ex]

$$s = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{(1+\sin\theta)^2 + (\cos\theta)^2}\, d\theta$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}$$

$$\sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1+\sin\theta}\, d\theta$$