Notes 12.4

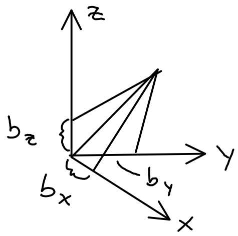
Thursday, March 22, 2007
8:32 PM

① $c\vec{a} = \langle ca_x, ca_y \rangle$

② $\vec{a} \cdot \vec{b}$ = scalar product

③ $\vec{a} \times \vec{b} = \vec{c}$ vector product

let $\vec{a} = \langle a_x, a_y, a_z \rangle$
$\vec{b} = \langle b_x, b_y, b_z \rangle$



$$\vec{a} \times \vec{b} = \langle \underbrace{a_y b_z - b_y a_z}_{\hat{i}}, \underbrace{a_z b_x - b_z a_x}_{\hat{j}}, \underbrace{a_x b_y - b_x a_y}_{\hat{k}} \rangle$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

ex) $\vec{a} = \langle 1,2,3 \rangle$ $\vec{b} = \langle 4,5,6 \rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & 2 \\ 4 & 5 \end{matrix}$$

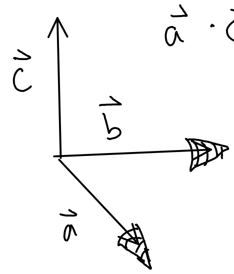
$$8\hat{k} + 15\hat{i} + 6\hat{j} - 12\hat{i} - 12\hat{j} - 5\hat{k} = 3\hat{i} - 6\hat{j} + 3\hat{k}$$
$$= \langle 3, -6, 3 \rangle$$

Property

- $\vec{a} \times \vec{a} = \vec{0}$

- If $\vec{a} \times \vec{b} = \vec{c}$ then $\vec{a} \perp \vec{c}$ & $\vec{b} \perp \vec{c}$

$\vec{a} \cdot \vec{c} = 0$ $\langle 1,2,3\rangle \cdot \langle 3,-6,3\rangle = 0$



- $\|\vec{a} \times b\| = \|\vec{a}\| \|\vec{b}\| \sin\theta$

or $\theta = \sin^{-1}\left(\frac{\|\vec{a} \times \vec{b}\|}{\|\vec{a}\| \|\vec{b}\|}\right)$

note if $\vec{a} \parallel \vec{b}$, $\theta = 0^\circ$, $\sin\theta = 0$
Test for $\parallel$ $\vec{a} \times \vec{b} = \vec{0}$ then $\vec{a} \parallel \vec{b}$

ex) #26) $(2,1,5)$ $(-1,3,4)$ $(3,0,6)$

$\vec{a}$ $\vec{b}$

$\vec{a} = \langle -3,2,-1\rangle$ $\vec{b} = \langle 4,-3,2\rangle$

$8\hat{k}$ $3\hat{i}$ $-6\hat{j}$

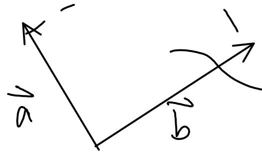
$$\vec{N} = \vec{a} \times \vec{b} = \left|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -1 \\ 4 & -3 & 2 \end{matrix}\right| \begin{matrix} \hat{i} & \hat{j} \\ -3 & 2 \\ 4 & -3 \end{matrix}$$

$-4\hat{i} + 4\hat{j} - 9\hat{k}$

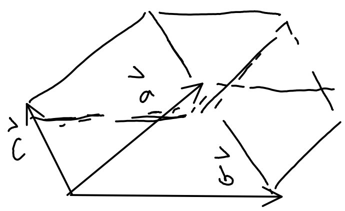
$= -\hat{i} - 2\hat{j} - \hat{k}$

$\langle -1,-2,-1\rangle$

On the plane possessing $\vec{a}$ and $\vec{b}$



$$\text{Area} = \|\vec{a} \times \vec{b}\|$$



$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$