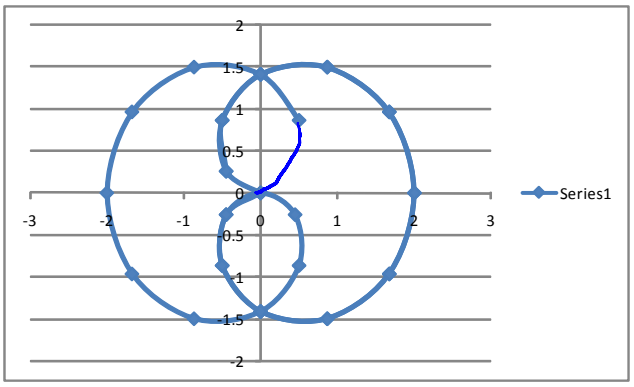
Take Home Quiz/Practice Test 10

Thursday, March 08, 2007
8:50 PM

① Sketch $r = 2\cos(\theta/2)$

theta	r	x	y
-3.14159	2.65359E-06	-2.6536E-06	-7.0415E-12
-2.61799	0.517640226	-0.448289 01	-0.258821 1
-2.09439	1.000001532	-0.499999 23	-0.866027 62
-1.5708	1.414214501	1.87637E-06	-1.414214 5
-1.0472	1.732051 25	0.866026952	-1.499999 62
-0.5236	1.931851767	1.673033134	-0.965925 14
0	2	2	0
0.523598	1.931851767	1.673033134	0.965925144
1.047197	1.732051 25	0.866026952	1.499999617
1.570795	1.414214501	1.87637E-06	1.414214501
2.094393	1.000001532	-0.499999 23	0.866027615
2.617992	0.517640226	-0.448289 01	0.258821104
3.14159	2.65359E-06	-2.6536E-06	7.0415E-12



② convert to polar coordinates $x^2 - y^2 = 1$

$r^2 = x^2 + y^2$ $\tan\theta = \frac{x}{y}$

$x = r\cos\theta$ $y = r\sin\theta$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$

$r^2 = \frac{1}{\cos 2x} \Rightarrow r = \frac{1}{\sqrt{\cos 2x}}$

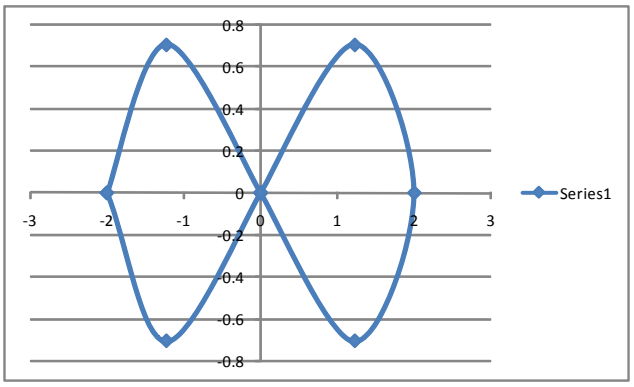
③ Convert to Cartesian coordinates $r\cos(\theta) = 1$

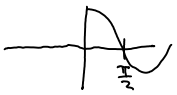
$x = r\cos\theta$ $x = 1$

④ Sketch the curve and find the area of $r^2 = 4\cos(2\theta)$

$r = \sqrt{4\cos(2\theta)}$

theta	r	x	y
-3.14159	2	-2	-5.3072E-06
-2.61799	1.414208146	-1.224738 62	-0.707106 78
-2.09439	#NUM!	#NUM!	#NUM!
-1.5708	#NUM!	#NUM!	#NUM!
-1.0472	#NUM!	#NUM!	#NUM!
-0.5236	1.414214646	1.224746122	-0.707106 78
0	2	2	0
0.523598	1.414214646	1.224746122	0.707106781
1.047197	#NUM!	#NUM!	#NUM!
1.570795	#NUM!	#NUM!	#NUM!
2.094393	#NUM!	#NUM!	#NUM!
2.617992	1.414208146	-1.224738 62	0.707106781
3.14159	$\frac{r^2}{2}d\theta$	-2	5.30718E-06



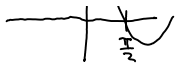


$2\theta = \frac{\pi}{2}$ $\theta = \frac{\pi}{4}$

$A = 4\int_0^{\frac{\pi}{4}} \frac{4\cos(2\theta)}{2} = 8\int_0^{\frac{\pi}{4}} \cos 2\theta$

	46	62	81
A=4159		-2	5.30718E-06

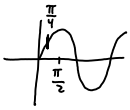
$A = \int \frac{r^2}{2} d\theta$



$2\theta = \frac{\pi}{2} \quad \theta = \frac{\pi}{4}$

$4\int_0^{\frac{\pi}{4}} \frac{4\cos(2\theta)}{2} = 8\int_0^{\frac{\pi}{4}} \cos 2\theta$

$4\sin 2\theta \Big|_0^{\frac{\pi}{4}}$



$4(\sin\frac{\pi}{2}) - 4(\sin 0)$

$= 4$

⑤ Find the arc length of the parameterized curve: $x = 3t^2$ $y = 2t^3$ for $t = 0$ to 2

$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ $\quad L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6t^2$

$L = \int_0^2 \sqrt{(6t)^2 + (6t^2)^2}\, dt$

$= \int_0^2 \sqrt{36t^2 + 36t^4}\, dt = \int_0^2 \sqrt{36t^2(1+t^2)}\, dt$

$= \int_0^2 6t\sqrt{1+t^2}\, dt$

let $u = 1 + t^2 \quad du = 2t\,dt$

$= \int_{t=0}^{t=2} 6t\sqrt{u}\, \frac{du}{2t} = 3\int_{t=0}^{t=2} \sqrt{u}\, du$

$= 3\left(2\frac{u^{3/2}}{3}\right)\Big|_{t=0}^{t=2}$

$2(1+t^2)^{3/2}\Big|_0^2 = 2(5)^{3/2} - 2(1)^{3/2}$

$2(5^{3/2} - 1)$