HW 11.1 #6,12,20,24,39,42

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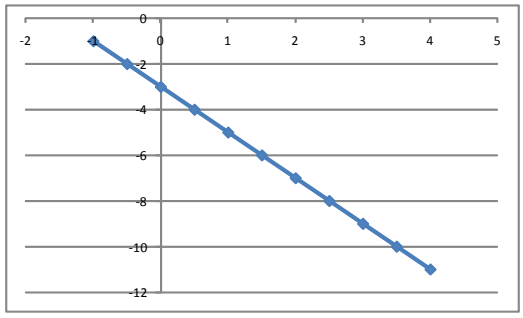
MATH 32A Section 1A

5–10

(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a Cartesian equation of the curve.

6. $x = 1 + t, \quad y = 5 - 2t, \quad -2 \le t \le 3$

(a)



t	x	y
-2	-1	-1
-1.5	-0.5	-2
-1	0	-3
-0.5	0.5	-4
0	1	-5
0.5	1.5	-6
1	2	-7
1.5	2.5	-8
2	3	-9
2.5	3.5	-10
3	4	-11

(b)

$t = x - 1$

$y = 5 - 2(x-1)$

$y = 5 - 2x + 2$

$y = -2x + 7$

11–18

(a) Eliminate the parameter to find a Cartesian equation of the curve.
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

12. $x = 4\cos\theta, \quad y = 5\sin\theta, \quad -\pi/2 \le \theta \le \pi/2$

(a)

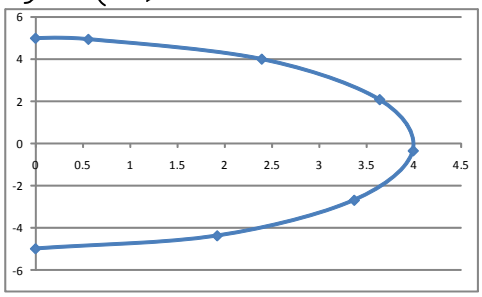
$\theta = \cos^{-1}\left(\frac{x}{4}\right)$

$y = 5\sin\left(\cos^{-1}\frac{x}{4}\right)$

$\left(\frac{x}{4}\right)^2 = \cos^2\theta \qquad \left(\frac{y}{5}\right)^2 = \sin^2\theta$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1 \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\cos^2\theta + \sin^2\theta = 1$

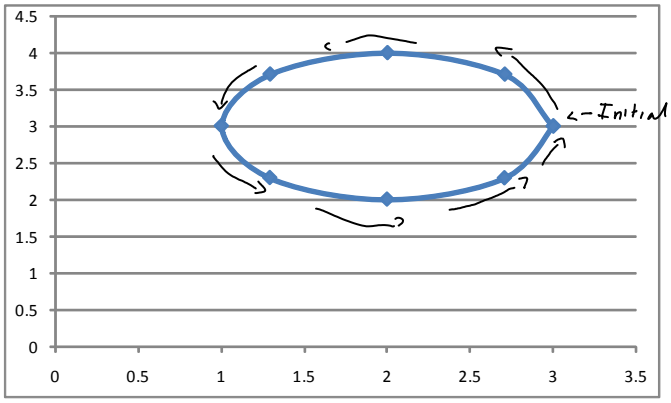


t	x	y
-1.57	0.003185	-5
-1.07	1.920497	-4.386
-0.57	3.367604	-2.69816
-0.07	3.990204	-0.34971
0.43	3.635863	2.084354
0.93	2.391336	4.0081
1.43	0.561326	4.950523
1.57	0.003185	4.999998

19–22 Describe the motion of a particle with position (x, y) as t varies in the given interval.

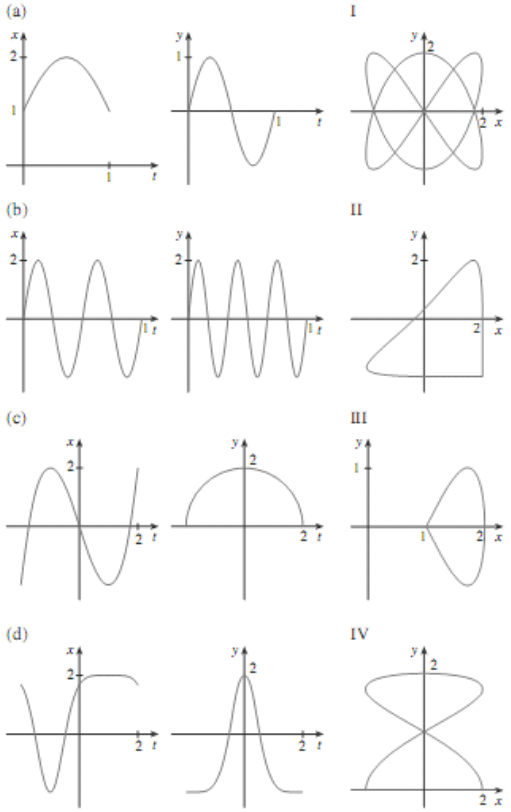
20. $x = 2 + \cos t, \quad y = 3 + \sin t, \quad 0 \le t \le 2\pi$

t	x	y
0	3	3
0.785	2.707388	3.706825
1.57	2.000796	4
2.355	1.293738	3.707951
3.14	1.000001	3.001593
3.925	1.291487	2.294302
4.71	1.997611	2.000003
5.495	2.705133	2.290925
6.28	2.999995	2.996815



When t=0 the initial point is (3,3). As t increases, the particle moves counterclockwise on the course of an ellipse. The terminal point when t=2π is back at (3,3).

24. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.



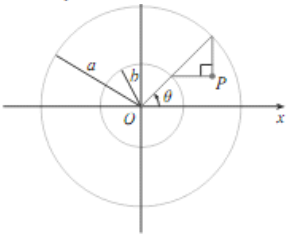
a. Answer: III
x starts at 0 and increases to 2 when t=.5. x then deceases to 1 between t=.5 and t=1.
y starts at 0 and increases to 1 when t=.25.

b. Answer: I
x starts at zero and oscillates with maximum and minimum values of 2 and -2. It repeats itself twice.
y is the same as x except it repeats 3 times.

c. Answer: IV
Looking at point t=0, x=0 and y=2. This is satisfied by answer IV

d. Answer: II
Looking at point t=0, x<2 and y=2. II satisfies this point.

39. If a and b are fixed numbers, find parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle θ as the parameter. Then eliminate the parameter and identify the curve.



$x = a\cos\theta$

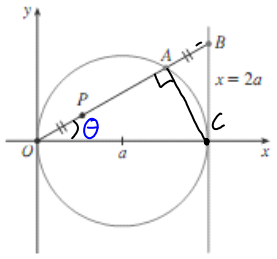
$y = b\sin\theta$

$x^2 = a^2\cos^2\theta$

$y^2 = b^2\sin^2\theta$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{ellipse}$$

42. Find parametric equations for the curve that consists of all possible positions of the point P in the figure, where $|OP| = |AB|$. Sketch the curve. (This curve is called the **cissoid of Diocles** after the Greek scholar Diocles, who introduced the cissoid as a graphical method for constructing the edge of a cube whose volume is twice that of a given cube.)



$$\cos\theta = \frac{2a}{\overline{OB}} \qquad \overline{OA} = 2a\cos\theta$$

$$\overline{OB} = \frac{2a}{\cos\theta}$$

$$\overline{OP} = \overline{AB} = \overline{OB} - \overline{OA} = 2a\left(\frac{1}{\cos\theta} - \cos\theta\right)$$

$$= 2a\left(\frac{1-\cos^2\theta}{\cos\theta}\right) = 2a\left(\frac{\sin^2\theta}{\cos\theta}\right) = 2a\sin\theta\tan\theta$$

$$x = 2a\sin\theta\tan\theta\cdot\cos\theta = 2a\sin^2\theta$$

$$y = 2a\sin\theta\tan\theta\cdot\sin\theta = 2a\sin^2\theta\tan\theta$$