HW 13.1 #6,8,14,16,40

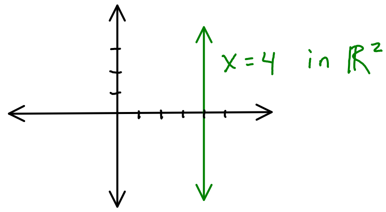
Monday, June 25, 2007
1:25 PM

Heather Graehl
MATH 32A Section 1A

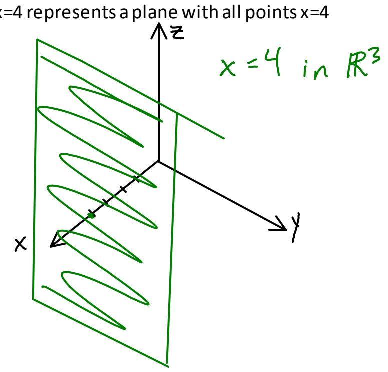
6. .

a) What does the equation x=4 represent in R^2? What does it represent in R^3? Illustrate with sketches

In R^2 x=4 represents a vertical line crossing through point (4,0)

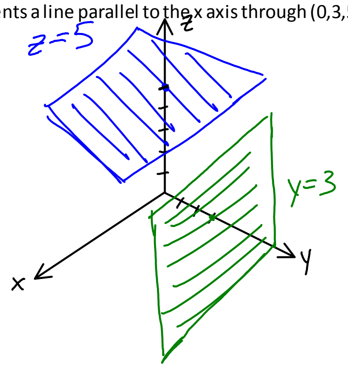


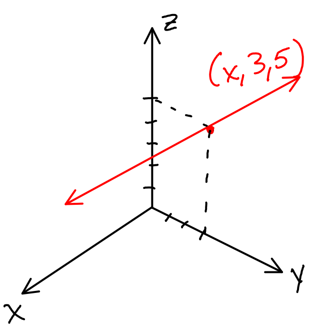
In R^3 x=4 represents a plane with all points x=4



b) What does the equation y=3 represent in R^3? What does z=5 represent? What does the pair of equations y=3 z=5 represent? In other words, describe the set of points (x,y,z) such that y=3 and z=5. Illustrate with a sketch.

y=3 in R^3 is a plane where y=3 at all points. z=5 also represents a plane where z=5 at all points. The pair y=3 and z=5 represents a line parallel to the x axis through (0,3,5).





8. Find the lengths of the sides of the triangle with vertices A(1,2,-3), B(3,4,-2), and C(3,-2,1). Is ABC a right triangle? Is it an isosceles triangle?

$D(A \& B) = \sqrt{4+4+1} = 3$

$D(A \& C) = \sqrt{(3-1)^2 + (-2-2)^2 + (1+3)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

$D(B \& C) = \sqrt{(3-3)^2 + (-2-4)^2 + (1+2)^2} = \sqrt{0+36+9} = \sqrt{45} = 3\sqrt{5}$

not isosceles

by pythag:
$9 + 36 \overset{?}{=} 45$ yes right triangle

14. Find an equation of the sphere that passes through the origin and whose center is (1,2,3).

$$\text{Dist } (0,0,0) \; \& \; (1,2,3) = r$$
$$\sqrt{1+2^2+3^2} = r$$
$$r = \sqrt{1+4+9} = \sqrt{14}$$
$$\boxed{(x-1)^2+(y-2)^2+(z-3)^2 = 14}$$

16. Show that the equation represents a sphere, and find its center and radius. $x^2 + y^2 + z^2 = 4x - 2y$

$$x^2-4x+y^2+2y+z^2=0$$
$$x^2-4x+(2)^2-(2)^2+y^2+2y+(1)^2-(1)^2+z^2=0$$
$$\boxed{\begin{array}{l}(x-2)^2+(y+1)^2+z^2 = 5\\ \text{center } (2,-1,0)\\ \text{radius } \sqrt{5}\end{array}}$$

40. Consider the points P such that the distance from P to A(-1,5,3) is twice the distance from P to B(6,2,-2). Show that the set of all such points is a sphere, and find its center and radius.

let $P = (x,y,z)$ D= distance

$$2 \times D(P \text{ to } B) = D(P \text{ to } A)$$
$$4[(x-6)^2+(y-2)^2+(z+2)^2]-[(x+1)^2+(y-5)^2+(z-3)^2]=0$$
$$4[x^2-12x+36+y^2-4y+4+z^2+4z+4]-[x^2+2x+1+y^2-10y+25+z^2-6z+9]=0$$
$$4x^2-48x+144+4y^2-16y+16+4z^2+16z+16-x^2-2x-1-y^2+10y-25-z^2+6z-9=0$$
$$3x^2-50x+3y^2-6y+3z^2+22z=-141$$
$$x^2-\frac{50}{3}x+y^2-2y+z^2+\frac{22}{3}z=\frac{-141}{3}$$
$$x^2-\frac{50}{3}+\left(\frac{50}{6}\right)^2-\left(\frac{50}{6}\right)^2+y^2-2y+(1)^2-(1)^2+z^2+\frac{22}{3}z+\left(\frac{22}{6}\right)^2-\left(\frac{22}{6}\right)^2=\frac{-141}{3}$$
$$\left(x-\frac{50}{6}\right)^2+(y-1)^2+\left(z+\frac{22}{6}\right)^2=\frac{-141}{3}+\frac{625}{9}+1+\frac{121}{9}=\frac{625}{9}+\frac{121}{9}+\frac{9}{9}-\frac{423}{9}$$
$$\left(x-\frac{50}{6}\right)^2+(y-1)^2+\left(z+\frac{22}{6}\right)^2=\frac{332}{9}$$
$$\boxed{\begin{array}{l}\text{center: } \left(\frac{50}{6}, 1, \frac{-22}{6}\right)\\ \text{radius: } \frac{\sqrt{332}}{3}\end{array}}$$