HW 13.3 #4,12,16,26,36,41

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MATH 32A Section 1A

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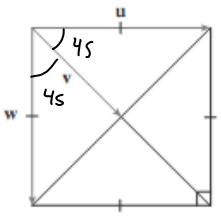
4. Find the dot product

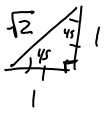
$a = \langle \frac{1}{2}, 4 \rangle \quad b = \langle -8, -3 \rangle$

$a \cdot b = (\frac{1}{2})(-8) + (4)(-3)$

$= -4 - 12 = \boxed{-16}$

12. If u is a unit vector, find $u \cdot v$ & $u \cdot w$





$\vec{u} = \langle 1, 0 \rangle \quad \vec{w} = \langle 0, 1 \rangle$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos(45°)$

$\vec{u} \cdot \vec{v} = 1 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{1}{2}}$

$u \cdot w = \boxed{0}$

16. Find the angle between the vectors. (first find an exact expression and then approximate to the nearest degree.)

$\vec{a} = \langle \sqrt{3}, 1 \rangle \quad b = \langle 0, 5 \rangle$

$\cos\theta = \frac{a \cdot b}{|\vec{a}||\vec{b}|}$

$\cos\theta = \frac{0+5}{\sqrt{4} \cdot \sqrt{25}}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \boxed{60°}$

26. For what vales of b are the vectors <-6,b,2> and <b,b^2,b> orthogonal?

$-6b + b^3 + 2b = 0$

$b^3 - 4b = 0$

$b(b^2 - 4) = 0$

$b(b+2)(b-2) = 0$

orthogonal when $\boxed{b = 0, 2, -2}$

36. Find the scalar and vector projections of b onto a.

$a = \langle 1, 2 \rangle \quad b = \langle -4, 1 \rangle$

$$\text{proj}_a b = \frac{a \cdot b}{|a|} = \frac{-4+2}{\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

$$\frac{-2}{\sqrt{5}} \frac{\langle 1,2 \rangle}{\sqrt{5}} = \left\langle \frac{-2}{\sqrt{5}}, \frac{-4}{\sqrt{5}} \right\rangle$$

41. Show that the vector $\text{orth}_a b = b - \text{proj}_a b$ is orthogonal to a. (it is called an orthogonal projection of b.)

$$\text{proj}_a b = \frac{a \cdot b}{|a|} \cdot \frac{\vec{a}}{|a|} = \frac{a \cdot b}{|a|^2} \cdot \vec{a}$$

$$b - \frac{a \cdot b}{|a|^2} \cdot \vec{a}$$