HW 13.6 #1, 6, 12, 21-28, 33, 35, 45

Heather Graehl
MATH 32A Section 1A

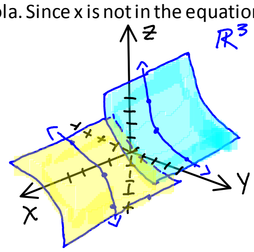
Monday, July 16, 2007
1:31 PM

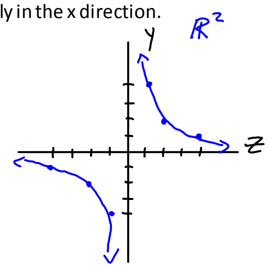
1. (a) What does the equation $y = x^2$ represent as a curve in $\mathbb{R}^2$?
(b) What does it represent as a surface in $\mathbb{R}^3$?
(c) What does the equation $z = y^2$ represent?

a) It represents a parabola opening up in the y direction
b) It represents a parabolic cylinder extending infinitely along the z direction
c) It is a parabolic cylinder just like in b, except it extends infinitely along the x direction

6. $yz = 4$ Describe and sketch

yz=4 forms a hyperbola. Since x is not in the equation, it extends infinitely in the x direction.





z	y
0	DNE
1	4
2	2
4	1
-1	-4
-2	-2
-4	-1

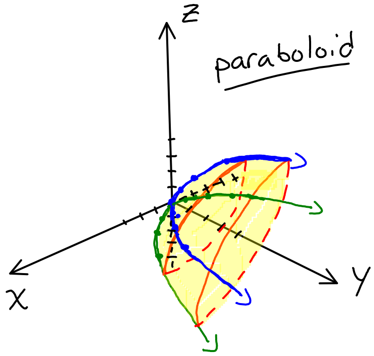
11–20 Find the traces of the given surface in the planes $x = k$, $y = k$, $z = k$. Then identify the surface and sketch it.

12. $4y = x^2 + z^2$

$\boxed{x=k}$ $z^2 = 4y$
$y = \frac{z^2}{4}$ parabola $y \geq 0$

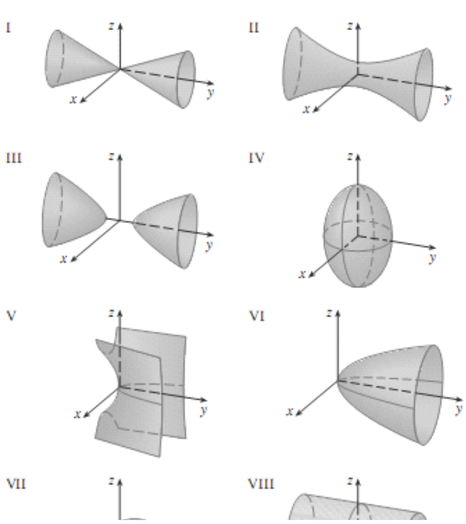
$\boxed{y=k}$ $4k = x^2 + z^2$ circle $k \geq 0$

$\boxed{z=k}$ $x^2 = 4y$
$y = \frac{x^2}{4}$ parabola $y \geq 0$



21–28 Match the equation with its graph (labeled I–VIII). Give reasons for your choices.

21. $x^2 + 4y^2 + 9z^2 = 1$
22. $9x^2 + 4y^2 + z^2 = 1$
23. $x^2 - y^2 + z^2 = 1$
24. $-x^2 + y^2 - z^2 = 1$
25. $y = 2x^2 + z^2$
26. $y^2 = x^2 + 2z^2$
27. $x^2 + 2z^2 = 1$
28. $y = x^2 - z^2$



21) $x=k$ $4y^2 + 9z^2 = 1 - k^2$ ellipse
$y=k$ $x^2 + 9z^2 = 1 - 4k^2$ ellipse
$z=k$ $x^2 + 4y^2 = 1 - 9k^2$ ellipse
possible answers: IV & VII
$\boxed{\text{Answer: VII}}$ because x int > z int.

22) $x=k$ $4y^2 + z^2 = 1 - 9k^2$ ellipse
$1 - 9k^2 \geq 0$
$-\frac{1}{3} = \frac{-1}{\sqrt{9}} \leq x \leq \frac{1}{\sqrt{9}} = \frac{1}{3}$
$y=k$ $9x^2 + z^2 = 1 - 4k^2$ ellipse
$z=k$ $9x^2 + 4y^2 = 1 - k^2$ ellipse
possible answers: IV & VII
$\boxed{\text{Answer: IV}}$ because z int > y int

23) $x=k$ $z^2 - y^2 = -k^2$ hyperbola
$y=k$ $x^2 + z^2 = k^2$ circle
$z=k$ $x^2 - y^2 = -z^2$ hyperbola
possible answers: I, II, III
I, II, III seem to be very similar except
I has an xz plane interception at (0,0,0),
III does not have an xz interception and II is continuous

III does not have an xz interception, and II is continuous at this region.

$\boxed{\text{Answer: II}}$

24) $-x^2+y^2-z^2=1$

possible answers: I, II, III

$x=k \quad y^2-z^2=1+k^2$ hyperbola

xz interception? $y=0 \quad -x^2-y^2=1 \rightarrow NO$

$\boxed{\text{answer: III}}$

$y=k \quad -x^2-z^2=1-k^2$

$x^2+z^2=k^2-1$ ellipse

$z=k \quad y^2-x^2=1+k^2$ hyperbola

25) $y=2x^2+z^2$

$\boxed{\text{Answer: VI}}$

$y=k \quad k=2x^2+z^2$ ellipse

$x=k \quad y=z^2+2k^2$ parabola

$z=k \quad y=2x^2+k^2$ parabola

26) $y^2=x^2+2z^2$

possible answers: I, II, III

$x=k \quad -k^2=z^2-y^2$ hyperbola

xz interception? $y=0 \quad 0=x^2+2z^2$

$y=k \quad k^2=x^2+2z^2$ ellipse

yes at $(0,0,0)$

$z=k \quad -2k^2=x^2-y^2$ hyperbola

$\boxed{\text{answer: I}}$

27) $x^2+2z^2=1$

ellipse extending infinitely in y direction

$\boxed{\text{answer: VIII}}$

28) $y=x^2-z^2$

$x=k \quad y=-z^2+k^2$ parabola (opening left)

$y=k \quad k=x^2-z^2$ hyperbola

$z=k \quad y=x^2-k^2$ parabola

$\boxed{\text{answer: V}}$

29–36 Reduce the equation to one of the standard forms, classify the surface, and sketch it.

33. $4x^2+y^2+4z^2-4y-24z+36=0$

$4x^2+y^2-4y+(2)^2+4(z^2-6z+(3)^2)+36-4(3)^2-(2)^2=0$

$4x^2+(y-2)^2+4(z-3)^2+36-36-4=0$

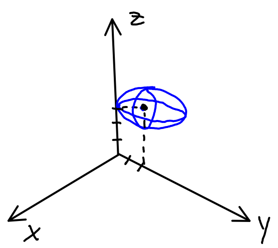
$\boxed{4x^2+(y-2)^2+4(z-3)^2-4=0}$

$x=k \quad (y-2)^2+4(z-3)^2=4$ ellipse

$y=k$ ellipse $\quad z=k$ ellipse

$\boxed{\text{forms an ellipsoid}}$

center $(0,2,3)$



35. $x^2-y^2+z^2-4x-2y-2z+4=0$

$x^2-4x+(2)^2-(y^2+2y+(1)^2)+z^2-2z+(1)^2+4-(2)^2-1-1=0$

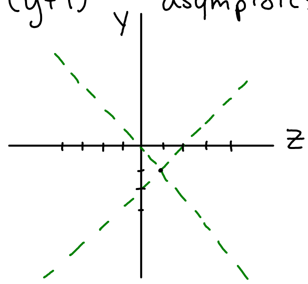
$\boxed{(x-2)^2-(y+1)^2+(z-1)^2-2=0}$

$x=k$ hyperbola $\quad (z-1)^2-(y+1)^2=2-(k-2)^2$

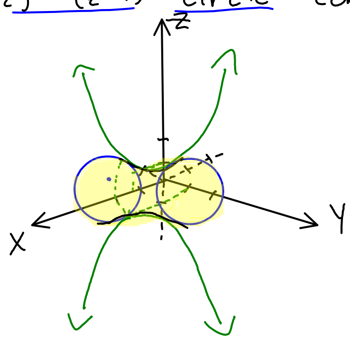
$y=k$ ellipse $\quad (x-2)^2+(z-1)^2=2+(y+1)^2$

$y=k$ ellipse $(x-2)^2+(z-1)^2 = 2+(y+1)^2$
$z=k$ hyperbola $(x-2)^2-(y+1)^2 = 2-(z-1)^2$

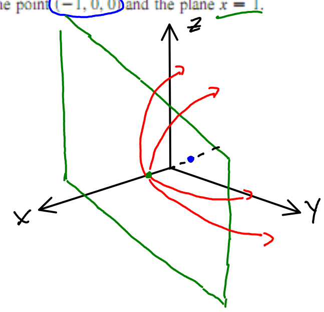
$x=k$ $(z-1)^2-(y+1)^2$ asymptotes $\pm\frac{b}{a} = \pm 1$ centered $z=1,\ y=-1$ $= (0,-1,1)$



$z=k$ $(x-2)^2-(y+1)^2$ asymptotes $\pm\frac{b}{a}=1$ centered $x=2,\ y=-1 = (2,-1,0)$
$y=k$ $(x-2)^2+(z-1)^2$ circle center $(2,0,1)$



45. Find an equation for the surface consisting of all points that are equidistant from the point $(-1, 0, 0)$ and the plane $x = 1$. Identify the surface.



let $Q = (x,y,z)$

Distance $(-1,0,0)$ & (x,y,z) $= \sqrt{(x+1)^2+(y-0)^2+(z+0)^2}$

Distance $x=1$ & $(x,y,z) = \frac{|x-1|}{\sqrt{1^2}} =$

$|x-1| = \sqrt{(x+1)^2+(y-0)^2+(z+0)^2}$
$(x-1)^2 = (x+1)^2+y^2+z^2$
$x^2-2x+1 = x^2+2x+1+y^2+z^2$
$\boxed{4x+y^2+z^2=0}$

$x=k$ $y^2+z^2 = -4k$ circle
$y=k$ $4x+z^2 = -k^2$ parabola
$z=k$ $4x+y^2 = -k^2$ parabola

$\boxed{\text{paraboloid}}$