HW 14.1 #1,6,16, 19-24, 34,39

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MATH 32A Section 1A

1–2 Find the domain of the vector function.

1. $\mathbf{r}(t) = \langle t^2, \sqrt{t-1}, \sqrt{5-t} \rangle$

$t^2 \to \mathbb{R}$

$t - 1 \geq 0 \to t \geq 1$

$5 - t \geq 0 \to t \leq 5$

$\boxed{1 \leq t \leq 5}$

6. $\lim_{t\to\infty} \left\langle \arctan t, e^{-2t}, \frac{\ln t}{t} \right\rangle$ Find the limit

$\lim_{t\to\infty} \frac{1}{\tan t} = \lim_{t\to\infty} \frac{\cos t}{\sin t} = \frac{\pi}{2}$

$\lim_{t\to\infty} e^{-2t} = \lim_{t\to\infty} \frac{1}{e^{2t}} = 0$

$\lim_{t\to\infty} \frac{\ln t}{t} = \frac{\infty}{\infty}$ LHR $\cdot \frac{\frac{1}{t}}{1} = 0$

$\lim_{t\to\infty} \left\langle \arctan(t), e^{-2t}, \frac{\ln t}{t} \right\rangle = \boxed{\left\langle \frac{\pi}{2}, 0, 0 \right\rangle}$

15–18 Find a vector equation and parametric equations for the line segment that joins P to Q.

16. $P(1, 0, 1)$, $Q(2, 3, 1)$

$\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1$

$\vec{r}(t) = (1-t)\langle 1,0,1 \rangle + t\langle 2,3,1 \rangle$

$\langle 1-t, 0, 1-t \rangle + \langle 2t, 3t, t \rangle$

$\langle 1+t, 3t, 1 \rangle$

<u>vector</u> $\vec{r}(t) = \langle 1+t, 3t, 1 \rangle \quad 0 \leq t \leq 1$

<u>parametric</u> $x = 1+t, \quad y = 3t, \quad z = 1$

19–24 Match the parametric equations with the graphs (labeled I–VI). Give reasons for your choices.

19. $x = \cos 4t, \quad y = t, \quad z = \sin 4t$

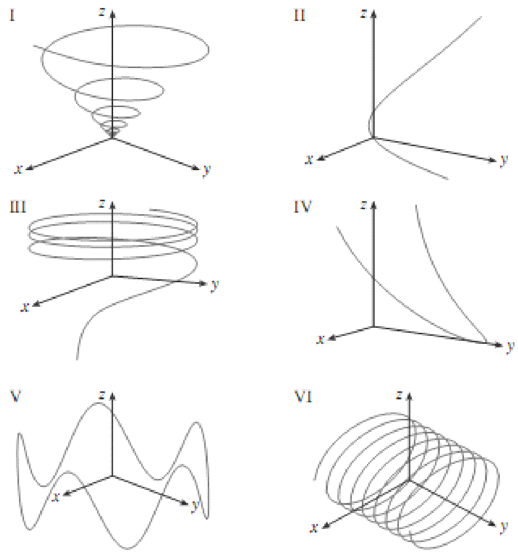
20. $x = t, \quad y = t^2, \quad z = e^{-t}$

21. $x = t, \quad y = 1/(1 + t^2), \quad z = t^2$

22. $x = e^{-t}\cos 10t, \quad y = e^{-t}\sin 10t, \quad z = e^{-t}$

23. $x = \cos t, \quad y = \sin t, \quad z = \sin 5t$

24. $x = \cos t, \quad y = \sin t, \quad z = \ln t$



19. When looking at the x- and z-axis, a circle is formed. Since y=t, it increases linearly which should create a spiral opening in the y-axis direction. Answer: VI

20. x-axis should linearly increase. y-axis will take the shape of a parabola. z axis is the inverse of an e^t graph. Answer: II

21. x-axis increases linearly. z-axis forms a parabola. Answer: IV

22. It is a spiral opening on the z axis. Answer: I

23. x- and y-axis alone form a circle. The z=sin will have oscillations in the z axis. Answer: V

24. x- and y-axis alone form a circle. Z axis of ln(t) causes it to spiral although it has strange behavior below t=1. Answer: III

34–36 Find a vector function that represents the curve of intersection of the two surfaces.

34. The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$

$x^2+y^2=4$ cyllinder

$\frac{z}{xy}=0$ plane

$x = 2\cos t$

$y = 2\sin t$

$\frac{1}{4}(x^2+y^2)=1$

$(2\cos t)^2 + (2\sin t)^2 = 4$

$\cos^2 t + \sin^2 t = 1$

$z = 2\cos t \cdot 2\sin t$

$\vec{r}(t) = \langle 2\cos(t), 2\sin(t), 4\cos(t)\sin(t) \rangle$

39. If two objects travel through space along two different curves, it's often important to know whether they will collide. (Will a missile hit its moving target? Will two aircraft collide?) The curves might intersect, but we need to know whether the objects are in the same position *at the same time*. Suppose the trajectories of two particles are given by the vector functions

$\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle \qquad \mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle$

for $t \geq 0$. Do the particles collide?

x parameter

$t^2 = 4t - 3$

$t^2 - 4t + 3 = 0$

$(t-1)(t-3) = 0$

$t = 1$ or 3

y parameter

z parameter

$7t - 12 = t^2$

$t^2 - 7t + 12 = 0$

$(t-3)(t-4) = 0$

$t = 3$ or 4

$t^2 = 5t - 6$

$t^2 - 5t + 6$

$(t-2)(t-3) = 0$

$t = 2$ or 3

$\boxed{\text{Yes, they collide at } t=3.}$