HW 14.4 #4,12,16,19,23,28,32

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MATH 32A Section 1A

3–8 Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t.

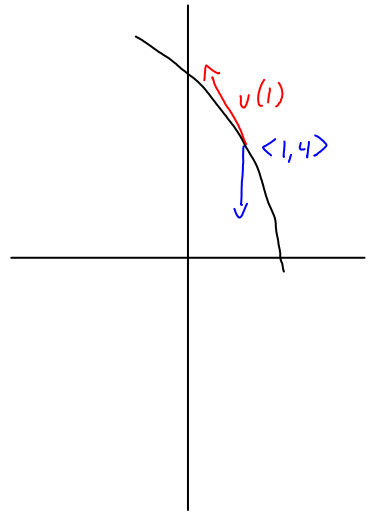
4. $\mathbf{r}(t) = \langle 2 - t, 4\sqrt{t} \rangle, \quad t = 1$

$r(t) = \langle 2-t, 4t^{1/2}$

$$\boxed{\text{velocity} = r'(t) = \langle -1, 2t^{-1/2} \rangle = \langle -1, 2 \rangle}$$

$$\boxed{\text{accelleration} = r''(t) = \langle 0, -t^{-3/2} \rangle = \langle 0, -1 \rangle}$$

$$\boxed{\text{Speed} = |r'(t)| = \sqrt{(-1)^2 + (2t^{-1/2})^2} = \sqrt{1 + \frac{4}{t}}}$$



9–14 Find the velocity, acceleration, and speed of a particle with the given position function.

12. $\mathbf{r}(t) = t^2\,\mathbf{i} + \ln t\,\mathbf{j} + t\,\mathbf{k}$

$r'(t) = \langle 2t, t^{-1}, 1 \rangle$ velocity
$r''(t) = \langle 2, -t^{-2}, 0 \rangle$ acceleration
$|r'(t)| = \sqrt{4t^2 + t^{-2} + 1}$ speed

15–16 Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.

16. $\mathbf{a}(t) = -10\,\mathbf{k}, \quad \mathbf{v}(0) = \mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \mathbf{r}(0) = 2\,\mathbf{i} + 3\,\mathbf{j}$

$v(t) = \int (-10k)dt = -10tk + c$
$i + j - k = v(0) = 0 + c \qquad c_1 = i + j - k$
$r(t) = \int (i + j - (10t + 1)k)dt = ti + tj - (5t^2 - t)k + c_2$
$2i + 3j = r(0) + c_2 \qquad c_2 = 2i + 3j$

$$\boxed{r(t) = (t+2)i + (t+3)j - (5t^2 + t)k}$$

19. The position function of a particle is given by $\mathbf{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$. When is the speed a minimum?

$v(t) = \langle 2t, 5, 2t - 16 \rangle$
$|v(t)| = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256} = \sqrt{8t^2 - 64t + 281}$
$\frac{d}{dt}|v(t)| = \frac{1}{2}(8t^2 - 64t + 281)^{-1/2} \cdot (16t - 64)$
$16t - 64 = 0$
$\boxed{t = 4}$

$\boxed{t=4}$

23. A projectile is fired with an initial speed of 500 m/s and angle of elevation 30°. Find (a) the range of the projectile, (b) the maximum height reached, and (c) the speed at impact.

(a) $|v(0)| = 500\,m/s$ $\qquad \frac{1}{2}(\sqrt{3}i+j)$

$v(0) = 250(\sqrt{3}i+j) = c_1$

$F(t) = -mgj \quad g = 9.8$

$a(t) = -gj$

$v(t) = -gtj + c$

$v(t) = 250\sqrt{3}ti + (250t - \frac{1}{2}gt^2)j = 0$

$t=0$ or $\frac{500}{g} = 51\,s$

$\boxed{\text{range } 250\cdot\sqrt{3}\cdot 51 = 22km}$

(b) $\frac{d}{dt}\left(250 - \frac{1}{2}gt^2\right) = 250 - gt = 0 \qquad t = \frac{250}{g} = 25.5$

$(250)(25.5) - g(25.5)^2\frac{1}{2} = \frac{(250)^2}{2g} = \boxed{3.2km}$

(c) $v\left(\frac{500}{g}\right) = 250\sqrt{3}i + \left(250 - g\left(\frac{500}{g}\right)\right)j = 250\sqrt{3}i - 250j$

$v\left(\frac{500}{g}\right) = 250\sqrt{3+1} = \boxed{500\,m/s}$

28. A batter hits a baseball 3 ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with speed 115 ft/s at an angle 50° above the horizontal. Is it a home run? (In other words, does the ball clear the fence?)

$v_0 = 115 ft/s \qquad \alpha = 50°$

$r(0) = 3j$

$r(t) = \frac{1}{2}gt^2j + tv_0 + D \qquad D = r(0) = 3j$

$v_0 = v_0\cos\alpha i + v_0\sin\alpha j$

$r(t) = (v_0\cos_\alpha)ti + \left((v_0\sin\alpha)t - \frac{1}{2}gt^2 + 3\right)j$

$x = v_0\cos\alpha t \qquad y = v_0\sin\alpha t - \frac{1}{2}gt^2 + 3$

$v_0\cos\alpha t = 400$

$t = \frac{400}{v_0\cos\alpha} = \frac{400}{115\cos 50°} = 5.41s$

$y = (115\sin 50°)(5.41) - \frac{1}{2}(32)(5.41)^2 + 3 = 11.2ft$

$\boxed{\text{yes, home run}}$

$\boxed{\text{yes, home run}}$

31–36 Find the tangential and normal components of the acceleration vector.

32. $\mathbf{r}(t) = (1 + t)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j}$

$r'(t) = i + (2t-2)j$

$|r'(t)| = \sqrt{1 + 4t^2 - 8t + 4} = \sqrt{4t^2 - 8t + 5}$

$r''(t) = 2j$

$r'(t) \times r''(t) = 2k$

$\dfrac{r'(t) \cdot r''(t)}{|r'(t)|} = \dfrac{2(2t-2)}{\sqrt{4t^2-8t+5}}$ tangent

$\dfrac{|r'(t) \times r''(t)|}{|r'(t)|} = \dfrac{2}{\sqrt{4t^2-8t+5}}$ normal