HW 15.2 #7, 11,16,24,30,33,36

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MATH 32A Section 1A

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5–20 Find the limit, if it exists, or show that the limit does not exist.

7. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$

• $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+mx} = \lim_{(x,y)\to(0,0)} \frac{x^2}{x(x+m)} = \lim_{(x,y)\to(0,0)} \frac{x}{x+m}$$

$$\boxed{\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} \quad \text{DNE}}$$

11. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

• $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{x^2m}{\sqrt{x^2+mx}} \cdot \frac{\sqrt{x^2+mx}}{\sqrt{x^2+mx}} = \frac{x^2m\sqrt{x^2+mx}}{x^2+mx} = \frac{xm\sqrt{x^2+mx}}{x+m}$$

$$\frac{xm\sqrt{x(x+m)}}{x+m}$$

squeeze theorem $\displaystyle 0 \le \lim_{(x,y)\to(0,0)} \frac{x\cdot y}{\sqrt{x^2+y^2}} \le \lim_{(x,y)\to(0,0)} \frac{xy}{x} = \boxed{0}$

16. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

• $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{x(mx)^4}{x^2+(mx)^8} = \frac{x^5m}{x^2(1+m^8x^6)} = \frac{x^3m}{1+m^8x^6}$$

• squeeze theorem $\displaystyle 0 \le \lim \frac{xy^4}{x^2+y^8} \le \frac{xy^4}{y^8} = \frac{x}{y^4}$

• $x = my^2$ $\displaystyle \lim \frac{my^2\cdot y^4}{(my^2)^2+y^8} = \lim \frac{my^6}{m^2y^4+y^8} = \lim \frac{my^6}{y^4(m^2+y^4)}$

• $x = my^4$ $\displaystyle \lim \frac{my^4\cdot y^4}{(my^4)^2+y^8} = \lim \frac{my^8}{m^2y^8+y^8} = \lim \frac{my^8}{y^8(m^2+1)} = \frac{m}{m^2+1}$

$$\boxed{\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^2} \quad \text{DNE}}$$

23–24 Find $h(x, y) = g(f(x, y))$ and the set on which h is continuous.

24. $g(t) = \dfrac{\sqrt{t}-1}{\sqrt{t}+1}$, $f(x, y) = x^2 - y$

$$g(f(x,y)) = \frac{\sqrt{x^2-y}-1}{\sqrt{x^2-y}+1}$$

$x^2 - y \ge 0$ $\qquad \sqrt{x^2-y} + 1 \ne 0$

$x^2 - y \geq 0$ $\quad \sqrt{x^2-y} + 1 \neq 0$

$x^2 \geq y$ $\quad \sqrt{x^2-y} \neq -1$ will never be 0

$$\boxed{\{(x,y) \mid x^2 \geq y\}}$$

27–36 Determine the set of points at which the function is continuous.

30. $F(x, y) = e^{x^2 y} + \sqrt{x + y^2}$

$x + y^2 \geq 0$

$$\boxed{\{(x,y) \mid x + y^2 \geq 0\}}$$

33. $f(x, y, z) = \dfrac{\sqrt{y}}{x^2 - y^2 + z^2}$

$y \geq 0$

$x^2 - y^2 + z^2 \neq 0$

$$\boxed{\{(x,y,z) \mid x^2 + z^2 \neq y^2,\ y \geq 0\}}$$

36. $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + xy + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+xy+y^2} = \lim \frac{xy}{(x+y)^2}$$

$\cdot\ y = mx \quad \lim \dfrac{x^2 m}{x^2 + x^2 m + (mx)^2}$

$$\lim \frac{x^2 m}{x^2(1+m+m^2)} = \frac{m}{1+m+m^2} \quad \text{DNE}$$

$$\boxed{\{(x,y) \mid (x,y) = (0,0)\}}$$