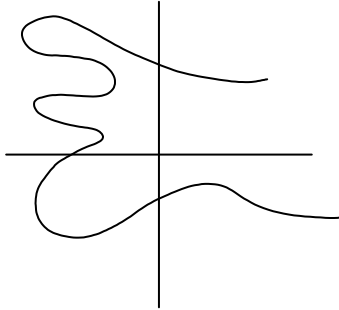


Notes 11.1

Tuesday, July 03, 2007
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Curves defined by parametric equations (11.1) in $\mathbb{R}^2$



~~$y=f(x)$~~ cannot be written as function

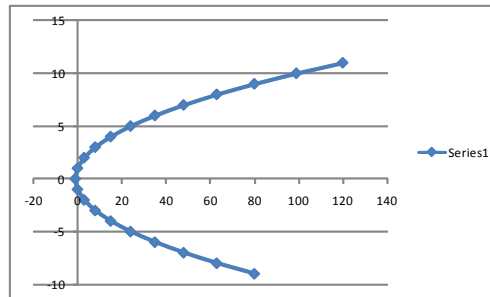
If a particle moves along this curve, its x and y coordinates can be written as functions of time

$$x=f(t), y=g(t)$$

A parameterization of the curve

Example: $x=t^2-2t$, $y=t+1$

t	x	y
-1	3	0
0	0	1
1	-1	2
2	0	3

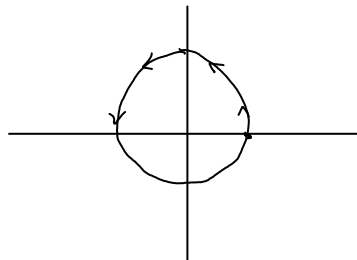


To check that it's a parabola, we eliminate the parameter. Here we solve for t .

$$\begin{aligned} y=t+1 &\Rightarrow t=y-1 \\ x=t^2-2t &\Rightarrow (y-1)^2 - 2(y-1) \\ &= y^2 - 2y + 1 - 2y + 2 \\ &= y^2 - 4y + 3 \quad \text{so this is a parabola} \end{aligned}$$

Example: $x=\cos(t)$, $y=\sin(t)$

t	x	y
0	1	0
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1

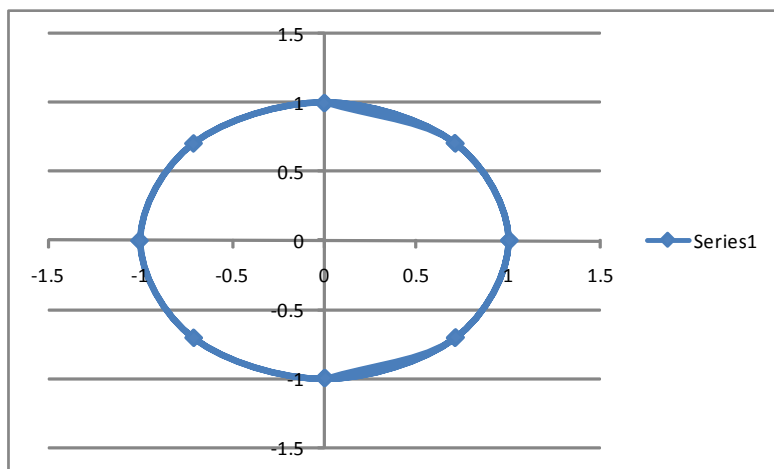


Is this a circle?

$$\begin{aligned} \sin^2 t + \cos^2 t &= 1 \\ x^2 + y^2 &= 1 \quad \text{yes} \end{aligned}$$

Example: $x=\sin(2t)$, $y=\cos(2t)$ $0 < t < 2\pi$

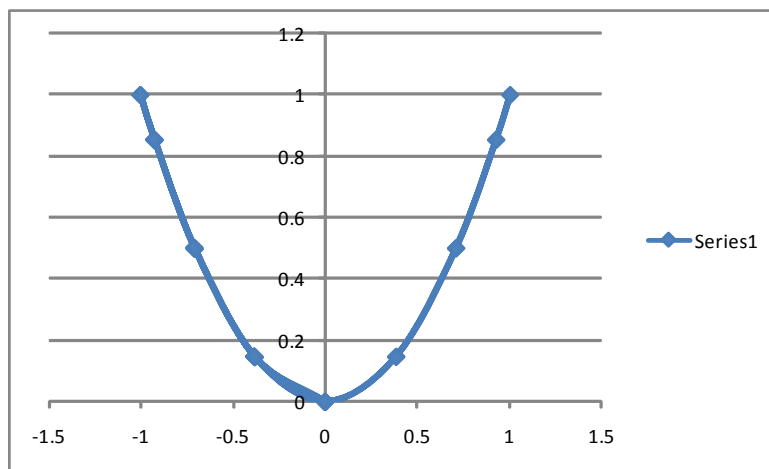
t	x	y
-3.14	0.003185	0.999995
-2.7475	0.709075	0.705133
-2.355	0.999997	-0.00239
-1.9625	0.705698	-0.70851
-1.57	-0.00159	-1
-1.1775	-0.70795	-0.70626
-0.785	-1	0.000796
-0.3925	-0.70683	0.707388
0	0	1
0.3925	0.706825	0.707388
0.785	1	0.000796
1.1775	0.707951	-0.70626
1.57	0.001593	-1
1.9625	-0.7057	-0.70851
2.355	-1	-0.00239
2.7475	-0.70907	0.705133
3.14	-0.00319	0.999995
3.5325	0.704568	0.709636
3.925	0.999992	0.003982
4.3175	0.710197	-0.704
4.71	0.004778	-0.99999
5.1025	-0.70344	-0.71076
5.495	-0.99998	-0.00557
5.8875	-0.71132	0.702871
6.28	-0.00637	0.99998



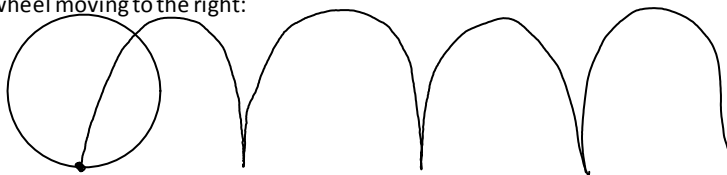
Example: $x=\sin(t)$, $y=\sin^2 t$

Eliminate the parameter
 $y=x^2$, but the actual parameter is cut off at (1,1)
and (-1,1)

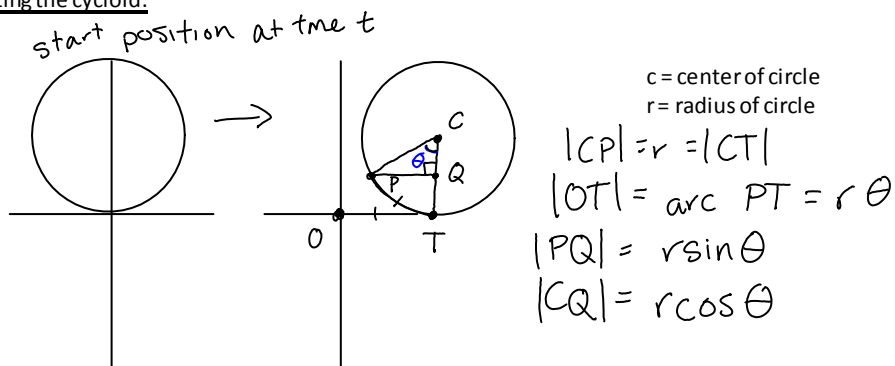
t	x	y
-3.14	-0.00159	2.53654E-06
-2.7475	-0.38397	0.147433385
-2.355	-0.70795	0.501194489
-1.9625	-0.92426	0.85425655
-1.57	-1	0.999999366
-1.1775	-0.92365	0.853130822
-0.785	-0.70683	0.499601837
-0.3925	-0.3825	0.146305865
0	0	0
0.3925	0.382499	0.146305865
0.785	0.706825	0.499601837
1.1775	0.923651	0.853130822
1.57	1	0.999999366
1.9625	0.92426	0.85425655
2.355	0.707951	0.501194489
2.7475	0.383971	0.147433385
3.14	0.001593	2.53654E-06
3.5325	-0.38103	0.145181934
3.925	-0.7057	0.498009188
4.3175	-0.92304	0.852001512
4.71	-1	0.999994293
5.1025	-0.92487	0.855378682
5.495	-0.70907	0.502787129
5.8875	-0.38544	0.148564483
6.28	-0.00319	1.01461E-05



Cycloid: traces a point on a wheel moving to the right:



Parameterizing the cycloid:



We'll parametrize with respect to θ

$$\begin{aligned} \text{x-coordinate of } P &= |OT| - |PQ| = r\theta - r\sin\theta \\ &= r(\theta - \sin\theta) \end{aligned}$$

$$\begin{aligned} \text{y-coordinate of } P &= |CT| - |CQ| = r - r\cos\theta \\ &= r(1 - \cos\theta) \end{aligned}$$

$$\text{so, } x = r(\theta - \sin\theta) \quad y = r(1 - \cos\theta)$$