Notes 13.2

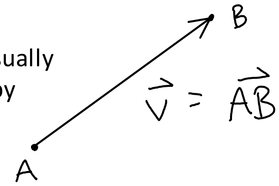
Tuesday, June 26, 2007
1:24 PM

<u>Vectors</u> - a vector is something that represents anything with a direction and magnitude (size).

Ex.

- A care traveling northeast at 50mph
- A man pushing a cart east with a force of 5N
- A particle moves from point A to point B.

Vectors are usually represented by arrows.

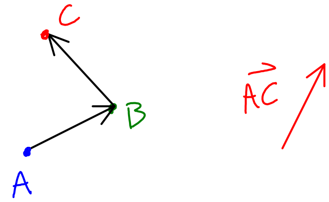


A is called the "initial point" or "tail"
B is called the "terminal point" or "tip"

*Note: It doesn't matter where you draw the vector, as long as it has the same length and direction.

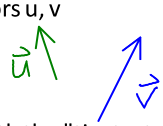
<u>Combining Vectors</u>

Lets say a particle moves from point A to point B, then from B to C.

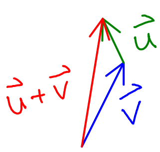


The vector $\vec{AC}$ is the end result is the end result. We define $\vec{AB} + \vec{BC} = \vec{AC}$

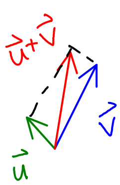
- In general, vectors $\vec{u}, \vec{v}$



We add them with the "tip-to-tail" method.



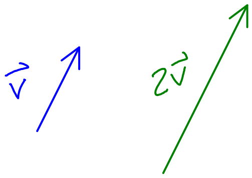
Or we can use parallelogram method. Draw $\vec{u} + \vec{v}$ starting from the tails, going to the intersection.



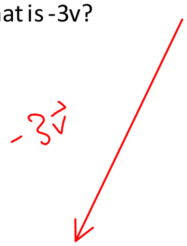
<u>How about multiplying?</u>

We can multiply a vector by a scalar (any real number).

What is $2\vec{v}$? $2\vec{v}$ has same direction as v but 2x the length.



What is -3v?



It is 3x as long as $\vec{v}$, but in the opposite direction.

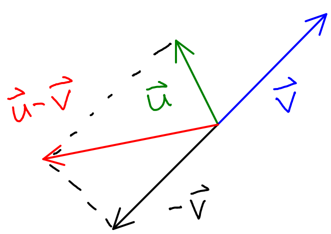
- Definition: If c is a real number, and $\vec{v}$ is a vector, then $c\vec{v}$ is the vector whose length is $|c|$ times the length of $\vec{v}$ and whose direction is:
  - Same as v, if c>0
  - Opposite, if c<0.
  - Zero vector if c=0 (no length or direction)
- Definition: The magnitude of a vector $\vec{v}$ is its length. It is denoted by $\|\vec{v}\|$ or $|\vec{v}|$

*NOTE: If we multiply a vector by -1, we set $(-1)\vec{v} = -\vec{v}$ It has magnitude $|\vec{v}|$, but opposite direction.

If $\vec{v} = \vec{AB}$, then $-\vec{v} = \vec{BA}$

Subtracting Vectors

Given vectors $\vec{u}, \vec{v}$, we define $\vec{u} - \vec{v}$ to be $\vec{u} + (-\vec{v})$.



If $\vec{u}$ and $\vec{v}$ have the same initial point, $\vec{u} - \vec{v}$ is the vector starting at the terminal point of $\vec{v}$, ending at the terminal point of $\vec{u}$.

Treating Vectors Algebraically

If we place the initial point of a vector at O (0,0,0), then its terminal point will be some $(a_1, a_2 a_3)$. We denote this vector by $\langle a_1, a_2, a_3 \rangle$.

a_1, a_2, a_3 are the components of the vector.

If a vector $\vec{v}$ starts at some $(a_1, a_2 a_3)$ and ends at $(b_1, b_2 b_3)$, then $\vec{v} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle$

Ex. What is the magnitude of $\vec{v}$ where $\vec{v} = \langle 1,-3,2 \rangle$

$$|\vec{v}| = \sqrt{1^2 + (-3)^2 + 2^2} = \sqrt{14}$$

In general, if a $\vec{v} = \langle a,b,c \rangle$, then $|\vec{v}| = \sqrt{a^2 + b^2 + c^2}$

How to Add Vectors Algebraically

$\vec{v} = \langle a_1, a_2, a_3 \rangle$
$\vec{b} = \langle b_1, b_2, b_3 \rangle$
c= scalar real number
$\vec{v} + \vec{w} = \langle a_1+b_1,\ a_2+b_2,\ a_3+b_3 \rangle$
$\vec{v} - \vec{w} = \langle a_1-b_1,\ a_2-b_2,\ a_3-b_3 \rangle$
$c\vec{v} = \langle ca_1,\ ca_2,\ ca_3 \rangle$

In $\mathbb{R}^3$, we have three important vectors:

1. $i = \langle 1,0,0 \rangle$
2. $J = \langle 0,1,0 \rangle$
3. $k = \langle 0,0,1 \rangle$

Why are these vectors important?

any vector $\vec{a} = \langle a_1, a_2 a_3 \rangle$ then $a = a_1 i + a_2 j + a_3 k$

Ex. $\vec{a} = 2i + 3j - 5k$
$\vec{b} = 4j - 7k$
Find $\vec{a} + \vec{b}$
$= 2i + 3j - 5k + 4j - 7k$
$= 2i + 7j - 12k$

Definition: A unit vector is a vector of length 1.

Ex.

i) i, j, k are unit vectors
ii) Let $\vec{v}$ be any non-zero vector. Then $\frac{1}{|\vec{v}|}\vec{v}$ is a unit vector. It's called the unit vector in the direction of $\vec{v}$.
iii) Find unit vector s in the diection of 2i - j -2k

$$|\vec{v}| = \sqrt{4+1+4} = 3$$

$$\frac{2}{3}i - \frac{1}{3}j - \frac{2}{3}k$$

Ex.

Forces: forces are represented by vectors.

A 100lb weight hangs from 2 wires: