Notes: 13.6 Cylinders and Quadric Surfaces

Tuesday, July 17, 2007
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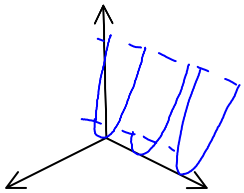
Graphing surfaces: quadric surfaces

To sketch a graph of a surface, we look at traces; i.e. the intersections of the surface with a plane $x=k$, $y=k$, $z=k$.

Cylinders: a cylinder is a surface that consists of all lines parallel to a given line and that pass through a given plane curve.

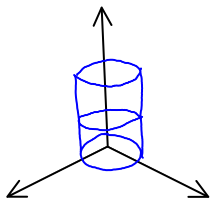
Example: $z=x^2$
What are the traces?
look at $y=k$



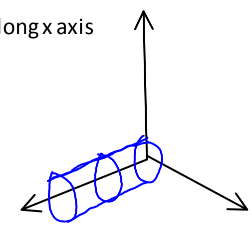
This is a cylinder
specifically a parabolic cylinder

NOTE: if you are missing a variable you have a cylinder.

Example: $x^2+y^2 = 1$
intersect with $z=k$ get a circle



Example: $y^2+z^2 = 1$
does not depend on x so stretches along x axis



Quadric Surfaces:

Surfaces for which the equations has all turns of degree at most 2.

e.g.

$3x^2 + 4z^2 + 5yz - 2 = 0$ quadric

$3x^2 + 4z^2 + 5yz^2 - 2 = 0$ not quadric surface

degree 3

These can be reduced to one of 2 standard forms:

$$\boxed{\begin{array}{c} Ax^2+By^2+Cz^2+D=0 \\ \text{or} \\ Ax^2+By^2+Cz=0 \end{array}}$$

Example: Use traces to sketch $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

Substitute $x=k$ $\quad k^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

$$\frac{y^2}{} + \frac{z^2}{} = 1-k^2 \quad \text{ellipse}$$

Substitute x=k

$$\frac{k^2}{1}+\frac{y^2}{9}+\frac{z^2}{4}=1$$

$$\frac{y^2}{9}+\frac{z^2}{4}=1-k^2 \quad \text{ellipse}$$

$$1-k^2 \geq 0$$

$$k^2 \leq 1$$

$$-1 \leq k \leq 1$$

So $x=2$ has no intersection

This surface is containted between the planes x=-1 and x=1

Substitute y=k

$$x^2+\frac{k^2}{9}+\frac{z^2}{4}=1$$

$$x^2+\frac{z^2}{4}=1-\frac{k^2}{9} \quad \text{ellipse}$$

$$1-\frac{k^2}{9} \geq 0$$

$$k^2 \leq 9$$

$$-3 \leq k \leq 3$$

This surface is contained between planes y= -3 and y= 3

Substitute z=k

$$x^2+\frac{y^2}{9}+\frac{k^2}{4}=1$$

$$x^2+\frac{y^2}{9}=1-\frac{k^2}{4} \quad \text{ellipse}$$

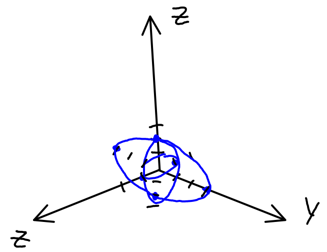
$$1-\frac{k^2}{4} \geq 0$$

$$k^2 \leq 4$$

$$-2 \leq k \leq 2$$

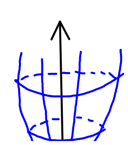
This surface is contained between planes of z = -2 and z = 2.

Now lets try to sketch:



Example: sketch $z=4x^2+y^2$

- $x=k$ $\quad z=4k^2+y^2$ ← parabola

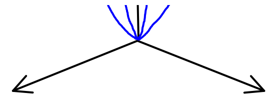


• $y=k$ $\quad z=4x^2+k^2$ ← parabola

thinner parabola

• $z=k$ $\quad k=4x^2+y^2$ ← ellipse

$k \geq 0$

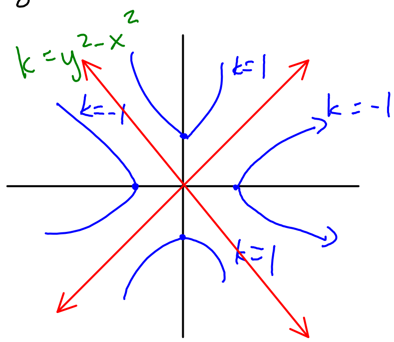


Example: sketch $z=y^2-x^2$

• $x=k$ $\quad z=y^2-k^2$ ← parabola

• $y=k$ $\quad z=k^2-x^2$ ← parabola

• $z=k$ $\quad k=y^2-x^2$ ← hyperbola

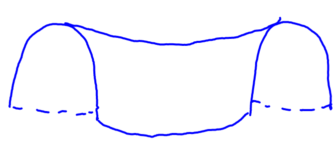


• $k=0$ $\quad 0=y^2-x^2$

$0=(y-x)(y+x)$

either $y=x$ or $y=-x$

This is a saddle:



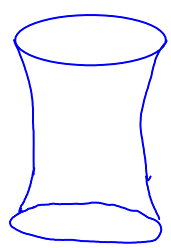
"hyperbolic paraboloid"

Example: Sketch $\frac{x^2}{4}+y^2-\frac{z^2}{4}=1$

• $x=k$ hyperbola $\quad -2 \leq x \leq 2$

• $y=k$ hyperbola $\quad -1 \leq y \leq 1$

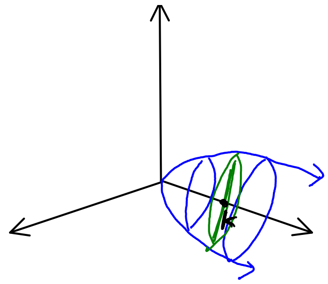
• $z=k$ ellipse



"Hyperboloid of one sheet"

Book has table on p872

Example: Find an equation for the surface given by rotating the parabola $y=x^2$ around the y axis.



• $y=k$ circle

center $=(0,k,0)$

in $x,z = (0,0)$

radius: $\sqrt{k}$

eq of circle: $x^2+z^2=k$

$$\boxed{y=x^2+z^2}$$