Notes: 15.1 Functions of Several Variables

Monday, July 16, 2007
1:27 PM

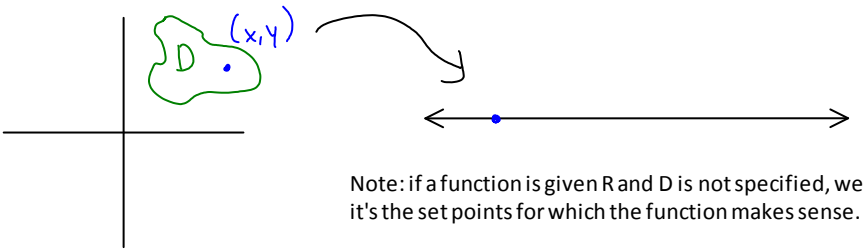
Question: What is a function?

- A function consists of 3 things:
  - Domain space
    - All the things you can plug into the function
  - Image space
    - outcome
  - Rule

- Ex: f(x)=sinx
  - Image space are real numbers, domain space are real numbers.
- In a function of several variables, the domain space will be R^2, R^3, R^n. The image space is still real numbers.

- Example: Volume of a cylinder is $V=\pi r^2 h$
  We think of this as a function of 2 variables.
  $V(r,h)=\pi r^2 h$
  Domain space is R^2
  Image space is R

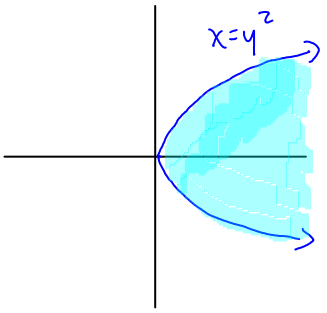
Definition: A function of 2 variables is a rule that assigns each ordered pair (x,y> in a set D a real number f(x,y).



Note: if a function is given R and D is not specified, we assume it's the set points for which the function makes sense.

Example: $f(x,y)=x\ln(x-y^2)$
Find the domain: $D:\{(x,y)\ x-y^2>0\}$

Sketch the domain:



Example: Wind Chill w(T,v) T=temperature V=wind speed

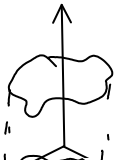
wind speed

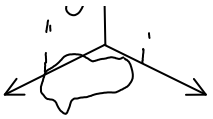
temp

v/t	5	10	15
5	4	3	2
0	-1	-3	-4
-5	-7	-9	-11

What is W(5,10)? What does it mean?
W(5,10)=3
This means that if the temperature is 5^0 C and the wind speed is 10 mph, ten it feels like it is 3^o C.

Graphs: If f is a function of 2 variables with domain D, then the graph of f is the set of ordered triples (x,y,z) in R^3 such that z=f(x,y) and((x,y)∈ |)





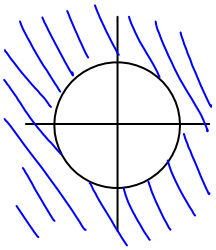
Example: $g(x,y) = \sqrt{9-x^2-y^2}$

1. Find and sketch domain
2. Find the range
3. Sketch the graph

1. $D: \{(x,y): 9-x^2-y^2 \geq 0\}$

$\Updownarrow$

$x^2+y^2 \leq 9$



2. Range $\{z: 0 \leq z \leq 3\}$

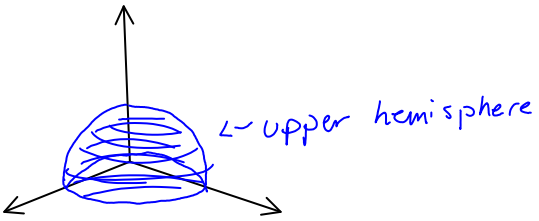
Is there (x,y) g(x,y)=0? $(0,3)$ $(0,0)$

3. Set $z = f(x,y)$

$z = \sqrt{9-x^2-y^2}$

$z^2 = 9-x^2-y^2$

$x^2+y^2+z^2 = 9$



Example: Sketch the graph of f(x,y) = 6 -3x -2y

$z = 6-3x-2y$

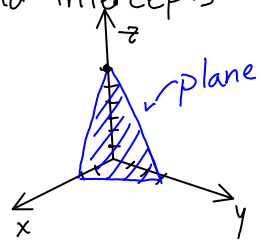
$3x+2y+z=6$

To graph planes find intercepts

x-int = (2,0,0)

y-int = (0,3,0)

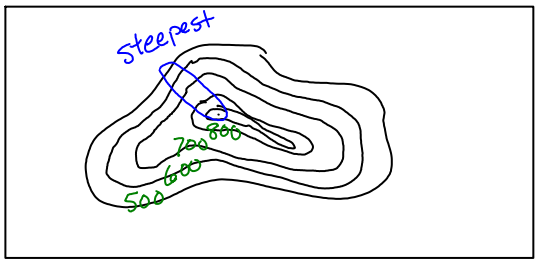
z-int = (0,0,6)



Level Curves:

Definition: The level curves of a function of 2 variables are the curves with equation f(x,y) = k, where k is a constant in the range of f.

Example: Topographical maps



Example: Sketch the level curves for f(x, y) = 6 - 3x - 2y for k = -6, 0, 6, 12

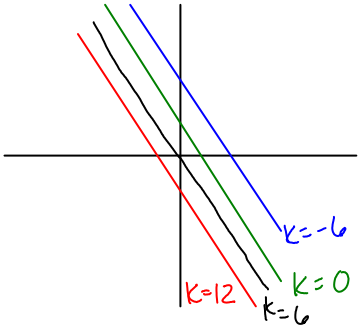
Definition says set f(x,y) = k

1. $6-3x-2y=-6 \Rightarrow 3x+2y=12$

2. $6-3x-2y=0 \Rightarrow$

$6-3x-2y=6$

$6-3x-2y=12$



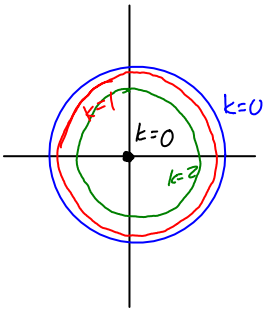
Example: sketch the level curves of $g(x,y)=\sqrt{9-x^2-y^2}$

$k=0,1,2,3$

$k=0 \quad \sqrt{9-x^2-y^2}=0$

$9-x^2-y^2=0$

$x^2+y^2=9$



$k=1 \quad x^2+y^2=8 \approx 2.828$

$k=2 \quad x^2+y^2=\sqrt{5} \approx 2.236$

$k=3 \quad x^2+y^2=0$

Functions of Three or More Variables:

Platonic solids

5 platonic solids in 3-D

$f(x,y,z)=\ln(z-y)+x\sqrt{y}\sin z$

What is domain?

$D=\{(x,y,z): z-y>0 \ \& \ y\geq 0\}$